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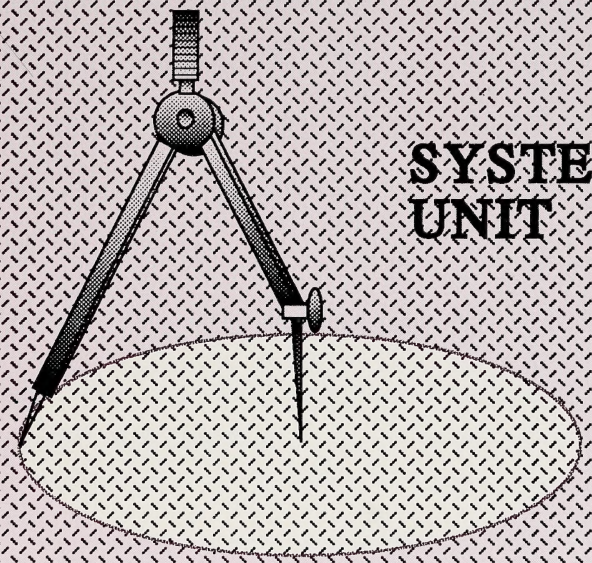
MATHEMATICS 23



SYSTEMS OF EQUATIONS
UNIT 5

Distance Learning

Alberta EDUCATION

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Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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Mathematics 23 Student Module Unit 5 Systems of Equations Alberta Distance Learning Centre ISBN No. 0-7741-0792-8 *1992

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General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

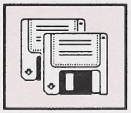
An explanation of what they mean is written beside each visual cue.

Key Idea
- flagging important ideas

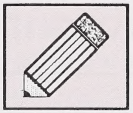
Audiotape
- learning by listening to an audiotape

What You Already Know
- reviewing what you already know

Another View
- exploring different perspectives



Computer Software
- learning by using computer software



Review
- studying previous concepts

Solutions
- correcting the activities

Videotape
- learning by viewing a videotape

Introduction
- introducing the unit

Extra Help
- providing additional study

Print Pathway
- choosing a print alternative

What Lies Ahead
- previewing the unit

Extensions
- going on with the topic

Calculator
- using your calculator

Exploring the Topic
- actively learning new concepts

What You Have Learned
- summarizing what you have learned

Mathematics 23

Course Overview

Mathematics 23 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Powers and Radicals	10%
Unit 2 Algebra	12%
Unit 3 Mathematics of Finance	4%
Unit 4 Linear Relations	12%
Unit 5 Systems of Equations	**16%**
Unit 6 Geometry	16%
Unit 7 Trigonometry	16%
Unit 8 Statistics	14%
	100%

Unit Assessment

After completing this unit you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal your teacher will determine what this assessment will be. It may be

Unit Assignment	- 50%
Supervised unit test	- 50%

Introduction to Systems of Equations

This unit covers topics dealing with Systems of Equations. Each topic contains explanations, examples and activities to assist you in understanding systems of equations. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the Solutions in **Appendix A**. In several cases there is more than one way to do the question.

Unit 5 Systems of Equations

Contents at a Glance

Value	**Systems of Equations**	
	What You Already Know	5
	Review	7
100%	**Topic 1:** Solving Systems of Equations • Introduction • Extra Help • What Lies Ahead • Extensions • Exploring Topic 1	9
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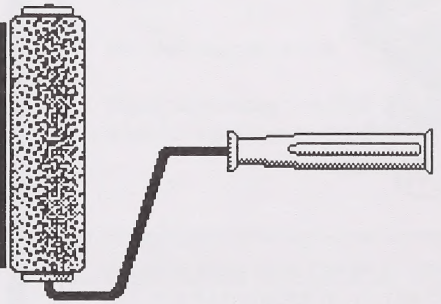
Systems of Equations

A system of equations is a set of equations that are to be treated together. In this unit, you will study systems having just two linear equations and two variables.

Suppose you were to think of two whole numbers having a sum of 7 and a difference of 3. The conditions the unknown numbers must meet can be represented by a system of equations.

Unit 5 Systems of Equations

Topic 1 Solving Systems of Equations



What You Already Know

Remember how to do the following.

1. Solve a linear equation in one variable.

$$y=\frac{3}{2}(y-8)+4$$

Multiply by 2. $2y=3(y-8)+8$

$2y=3y-24+8$

Subtract $3y$. $-y=-24+8$

$-y=-16$

$y=16$

2. Isolate one of the variables in a linear equation having two variables.

$4x+2y=5$

Subtract $4x$. $2y=-4x+5$

Divide by 2. $y=-2x+\frac{5}{2}$

3. Solve a problem using a linear equation.

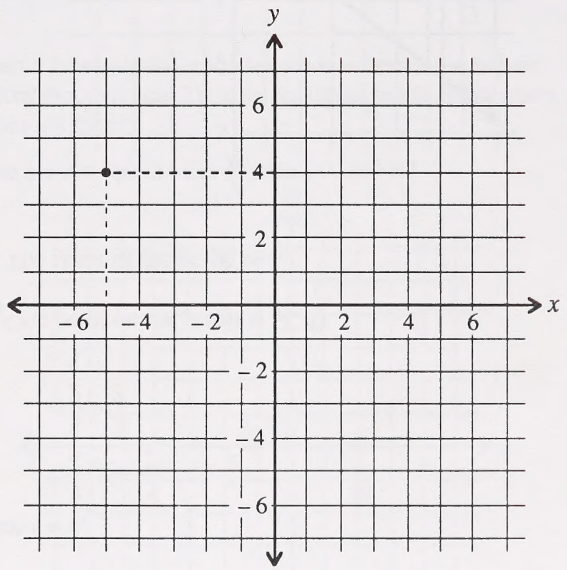
The sum of 11 and nine times a number is 56.
What is the number?
Let the number be n.
Then, $9n+11=56$

$9n=45$

$n=5$.

The number is 5.

4. Identify the coordinates of a point.

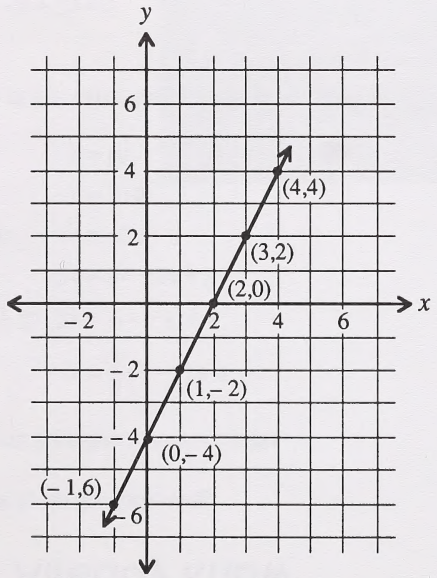


For the point placed on the Cartesian plane, $x=-5$ and $y=4$.

5. Graph a linear equation using a table of values.

$y = 2x - 4$.

x	y
-1	-6
0	-4
1	-2
2	0
3	2
4	4



6. Graph a linear equation using a point and the slope.

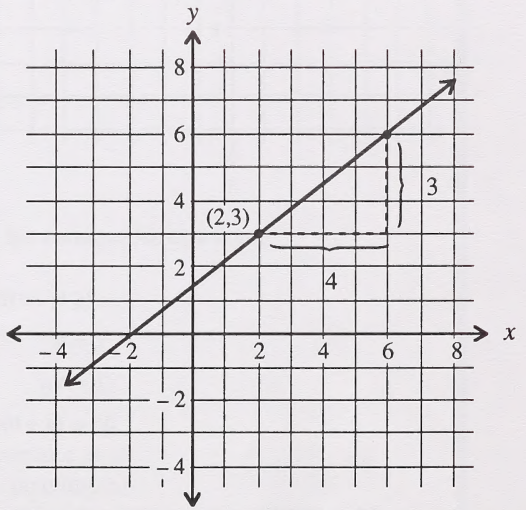
$$y = \frac{3}{4}x + \frac{3}{2}.$$

When $x = 2$,

$$y = \frac{3}{4}(2) + \frac{3}{2}$$
$$= \frac{3}{2} + \frac{3}{2}$$
$$= 3.$$

One point on the graph is (2, 3).

The slope of the graph is $\frac{3}{4}$.



7. Graph a linear equation using the intercepts.

$5x - 7y = -35.$

When $x = 0$,

$5(0) - 7y = -35.$

$-7y = -35$

$y = 5$

The y-intercept is (0, 5).

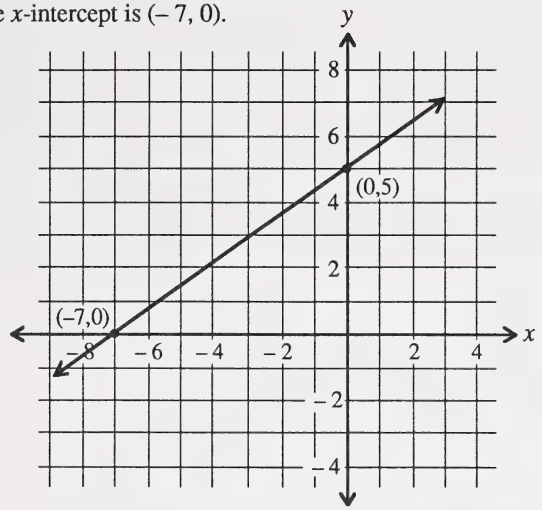
When $y = 0$,

$5x - 7(0) = -35.$

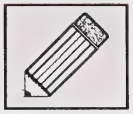
$5x = -35$

$x = -7$

The x-intercept is (– 7, 0).



Now that you have looked at material that you studied previously, go to the **Review** to confirm your understanding of this material.



Review

Try the following review questions. When needed, turn to Appendix B for graph paper.

1. Solve the equation.

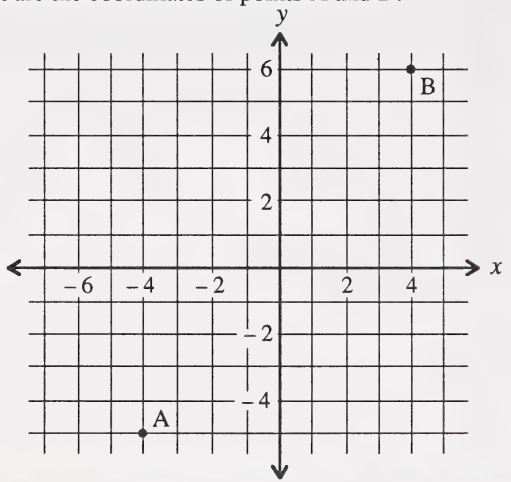
   $5x - 10 + 2x = 60$

2. Solve for y.

   $2x - 3y = -12$

3. When a farmer looks at the horses in a corral, the farmer notices that there are 21 more legs than heads. How many horses are there?

4. What are the coordinates of points A and B?



5. Use a table of values to graph

   $y = \frac{2}{3}x + 1.$

6. Use a point and the slope to graph

   $y = \frac{4}{5}x + \frac{3}{5}.$

7. Use the intercepts to graph

   $8x + 5y = 40.$

Now go to the **Review** solutions in **Appendix A**.

If you had any trouble with questions 1, 2, and 3 go to Mathematics 23, Unit 2.

If you had any trouble with questions 4, 5, 6, and 7 go to Mathematics 23, Unit 4.

Topic 1 Solving Systems of Equations

Introduction

In order to solve a system of two equations in two variables, you must find the value of each of the two variables in order to make both equations true.

Think about the system representing the conditions that the sum of two whole numbers must be 7 and their difference must be 3. You have solved the system when you have found two numbers that satisfy the conditions.

You may have been able to solve the problem using trial and error. This may not always be feasible. In this topic, you will learn how to solve systems of equations using graphs and algebraic methods.

What Lies Ahead

Throughout the topic you will learn to

1. solve systems of equations graphically

2. solve systems of equations algebraically

Now that you know what to expect, turn the page to begin your study of solving systems of equations.

Exploring Topic 1

Activity 1

Solve systems of equations graphically.

The following are systems of equations:

$$\text{A}\begin{cases}3x-2y=6\\-2x+3y=-9\end{cases} \qquad \text{B}\begin{cases}x^2+y=4\\x=y+1\end{cases} \qquad \text{C}\begin{cases}x+y+z=3\\2x+3y+z=2\\x+y+4z=9\end{cases}$$

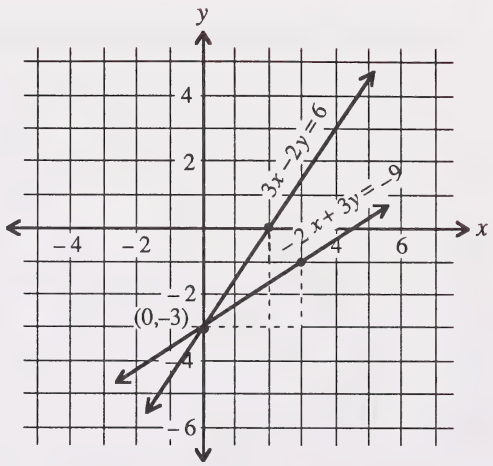
In this topic, you will limit your study to systems like System A. In System B, one of the variables is raised to the second power. In System C, there are more than two equations and more than two variables. Therefore, Systems B and C are beyond the scope of this topic.

To solve a system of two equations is to find the values of the variables that make both equations true. This can be done by determining the coordinates of the point of intersection of the two graphs of the equations of the system.

You can graph the equations of System A by putting them into the slope-intercept form.

$3x-2y=6$ becomes $y=\frac{3}{2}x-3$.

$-2x+3y=-9$ becomes $y=\frac{2}{3}x-3$.



The point of intersection is $(0, -3)$.
The solution of System A is $x = 0$ and $y = -3$.
A solution of a system may be expressed as an ordered pair; you can say that the solution of System A is $(0, -3)$.
Just to be sure, you should substitute these values back into each equation of the system to see that each equation is satisfied.
Check: $x = 0$, $y = -3$

LS	RS
$3x-2y$	6
$3(0)-2(-3)$	
$0+6$	
6	
LS = RS	

LS	RS
$-2x+3y$	-9
$-2(0)+3(-3)$	
$0-9$	
-9	
LS = RS	

The solution is correct.

Now go back to the problem of finding two whole numbers having a sum of 7 and a difference of 3. The conditions can be represented by two equations.

Let x be the larger number and y the smaller.
Then, $x + y = 7$ and $x - y = 3$. The system to be solved can be written as

$x+y=7$ (1)
$x-y=3$ (2)

(Equations may be numbered to make it easier to refer to them.)

One way to graph the equations is to use their x- and y-intercepts.

For (1): When $x = 0$,

$x+y=7$
$0+y=7$
$y=7$ The y-intercept is $(0,\ 7)$.

When $y = 0$,

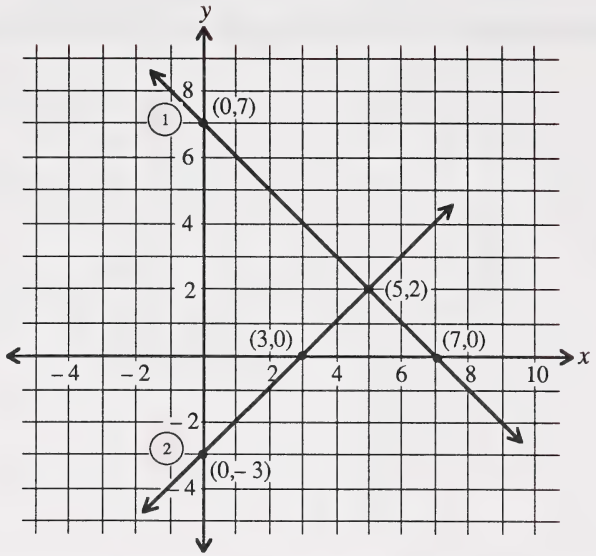
$x+y=7$
$x+0=7$
$x=7$ The x-intercept is $(7,\ 0)$.

For (2): When $x = 0$,

$x-y=3$
$0-y=3$
$y=-3$ The y-intercept is $(0,\ -3)$.

When $y = 0$,

$x-y=3$
$x-0=3$
$x=3$ The x-intercept is $(3,\ 0)$.



The graphs intersect at (5, 2), so the solution of the system is $x = 5$ and $y = 2$.

You can verify this.

Check: $x = 5, y = 2$

LS	RS
$x + y$	7
$5 + 2$	
7	

LS = RS

LS	RS
$x - y$	3
$5 - 2$	
3	

LS = RS

The two numbers are 5 and 2. Their sum is 7 and their difference is 3 as required.

You can see another application of systems of equations in determining the dimensions of a certain rectangle. The rectangle had a perimeter of 36 cm and a length that is twice the width.

Let x cm be the width and y cm be the length. The perimeter condition gives you $2x + 2y = 36$. The formula $y = 2x$ reflects the condition that the length is twice the width.

The system to be solved is

$$2x+2y=36 \quad (1)$$
$$y=2x \quad (2)$$

For (1): When $y = 0$,

$$2x+2y=36$$
$$2x+0=36$$
$$x=18$$

The x-intercept is (18, 0).

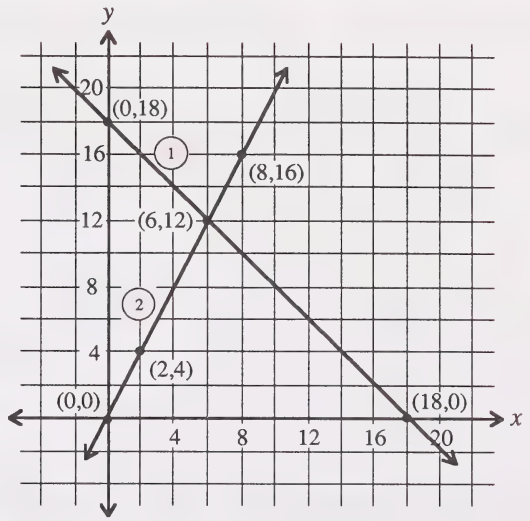
When $x = 0$,

$$2x+2y=36$$
$$0+2y=36$$
$$y=18$$

The y-intercept is (0, 18).

For (2): For $y = 2x$,

x	y
0	0
2	4
8	16



The graphs cross at (6, 12). The solution of the system is $x = 6$ and $y = 12$. This can be verified.

Check: $x = 6$, $y = 12$

LS	RS
$2x + 2y$	36
$2(6) + 2(12)$	
$12+24$	
36	

LS = RS

LS	RS
y	$2x$
12	$2(6)$
	12

LS = RS

The rectangle has a length of 12 cm and a width of 6 cm.

$$P = 2(l + w)$$
$$= 2(12\text{ cm} + 6\text{ cm})$$
$$= 2(18\text{ cm})$$
$$= 36\text{ cm}$$

The perimeter of the rectangle is 36 cm, as it should be.
The length, 12 cm, is twice the width as required.

You have seen how systems of equations can be used. You can now focus on solving the systems themselves.

Look at the following examples.

Example 1

Solve the system of equations and check your answer.

$$y = 2x + 4 \quad (1)$$
$$x + y = 10 \quad (2)$$

Solution:

For (1), the y-intercept is (0, 4) and the slope is 2.

For (2): When $x = 0$,

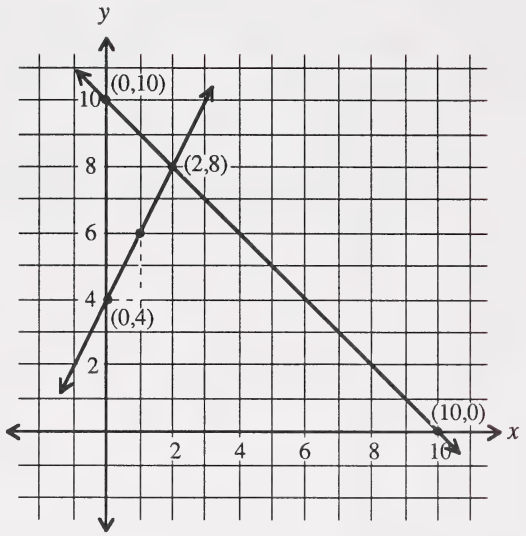
$$x + y = 10$$
$$(0) + y = 10$$
$$y = 10$$

The y-intercept is (0, 10).

When $y = 0$,

$$x + y = 10$$
$$x + (0) = 10$$
$$x = 10$$

The x-intercept is (10, 0).



The solution of the system is $x = 2$ and $y = 8$ or (2, 8).

Check: $x = 2$, $y = 8$

LS	RS
y	$2x + 4$
8	$2(2) + 4$
	8

LS = RS

LS	RS
$x + y$	10
$2 + 8$	
10	

LS = RS

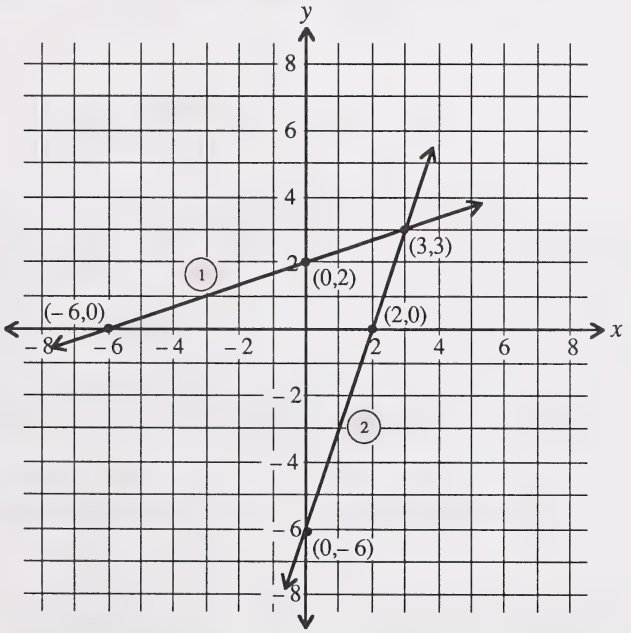
Example 2

Solve the system of equations and check your solution.

$x - 3y = -6$ (1)
$3x - y = 6$ (2)

Solution:

For (1), the intercepts are (– 6, 0) and (0, 2). For (2), the intercepts are (0, – 6) and (2, 0).



The intersection of the graphs is (3, 3).

The solution of the system is $x = 3$ and $y = 3$ or (3, 3).

Check: $x = 3, y = 3$

LS	RS
$x - 3y$	-6
$3 - 3(3)$	
-6	

LS = RS

LS	RS
$3x - y$	6
$3(3) - 3$	
6	

LS = RS

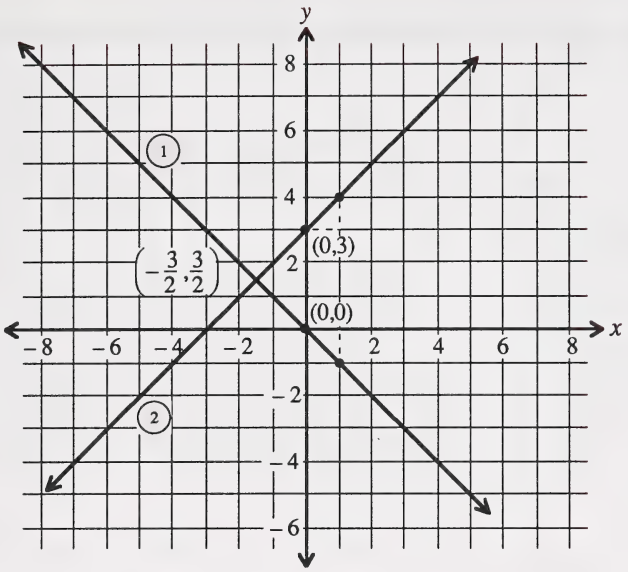
Example 3

Solve the system of equations and check your solution.

$x = -y$ (1)
$y = x + 3$ (2)

Solution:

In the slope-intercept form, (1) becomes $y = -x$.
The y-intercept is (0, 0) and the slope is – 1.
For (2), the slope is 1 and the y-intercept is (0, 3).



The intersection of the graphs is $\left(-\frac{3}{2},\ \frac{3}{2}\right)$.

The solution of the system is $x=-\frac{3}{2}$ and $y=\frac{3}{2}$.

Check: $x=-\frac{3}{2},\ y=\frac{3}{2}$

LS	RS
x	y
$-\frac{3}{2}$	$-\left(\frac{3}{2}\right)$
	$-\frac{3}{2}$

LS = RS

LS	RS
y	$x+3$
$\frac{3}{2}$	$\left(-\frac{3}{2}\right)+3$
	$\frac{3}{2}$

LS = RS

Try the following questions. Do the odd-numbered questions. If you require additional practice, do the rest of the questions. Turn to **Appendix B** for graph paper. Solve the systems of equations by using graphs. You may use any one of a variety of ways to graph the linear equations. Make your graphs from a table of values, two points, or the slope and y-intercept. Check your answers.

1. $x + y = 7$ (1)
   $x - y = 1$ (2)

2. $x - 3y = -6$ (1)
   $2x + 3y = 15$ (2)

3. $x - 4y = -20$ (1)
   $2x + y = -4$ (2)

4. $x + 2y = 14$ (1)
   $y = 2x + 2$ (2)

5. $y = 2x + 5$ (1)
   $x = y$ (2)

6. $x + 2y = -10$ (1)
   $x - 4y = 32$ (2)

For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.

So far you have found that systems of equations have one solution. This is not always the case.

Try the following question in which you are to solve the system using a graph. Turn to **Appendix B** for graph paper.

7. $2y - 3x = 6$ (1)
   $3x - 2y = 12$ (2)

For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.

Hopefully, you didn't try too hard to find a solution for the system; there isn't any. In graphing the equations, you would have discovered that their graphs were parallel and do not intersect. There isn't an ordered pair that belongs to both equations.

The systems of equations in the next question illustrate another interesting case.

Try the following question in which you are to solve the system using a graph. Turn to Appendix B for graph paper.

8. $y = -2x - 4$ (1)
   $4x + 2y = -8$ (2)

For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.

When you graphed the equations, you found that their graphs coincided. This means they have exactly the same graphs. An ordered pair that belongs to one equation belongs to the other. For example, $(-1, 2)$, $(0, -4)$, and $(2, -8)$ belong to both equations. Since each equation has an infinite number of solutions, the system of equations had an infinite number of solutions too.

When the equations of a system are put into the slope-intercept form, it is easy to see whether the system has one solution, no solution, or an infinte number of solutions.

Suppose a system can be represented as follows.

$$y = m_1 x + b_1$$
$$y = m_2 x + b_2$$

If $m_1 = m_2$, there are two possibilities, depending on the y-intercepts. If $b_1 = b_2$, the system has an infinite number of solutions. If $b_1 \neq b_2$, the system has no solutions.

If $m_1 \neq m_2$, the system has exactly one solution (regardless of whether $b_1 \neq b_2$ or $b_1 = b_2$).

Try the following questions. Represent the equations of the systems in the slope-intercept form. By comparing slopes and y-intercepts, determine whether the system has one solution, an infinite number of solutions, or no solutions.

9. $x - 4y = -20$ (1)
   $2x + y = -4$ (2)

10. $2y - 3x = 6$ (1)
    $3x - 2y = 12$ (2)

11. $y = -2x - 4$ (1)
    $4x + 2y = -8$ (2)

For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.

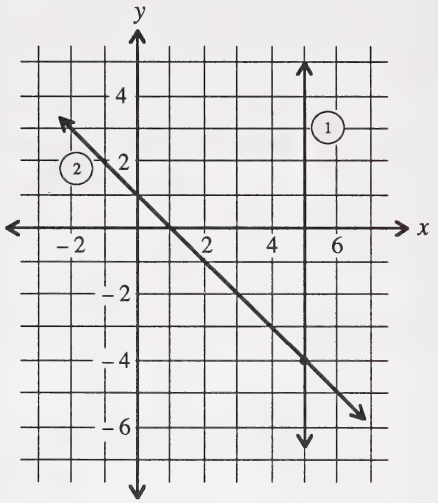
In some systems, it may not be possible to put equations into the $y = mx + b$ form.

$x = 5$ (1)
$x + y = 1$ (2)

Equation (1) cannot be put into the $y = mx + b$ form.

Equation (2) can. $y = -x + 1$.

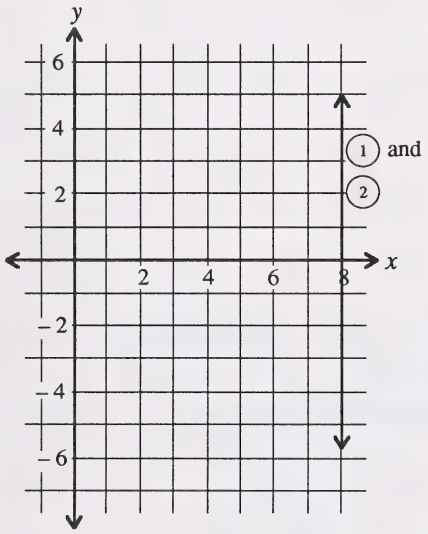
When one of the equations but not the other can be put into the $y = mx + b$ form, the system has exactly one solution.



In the system

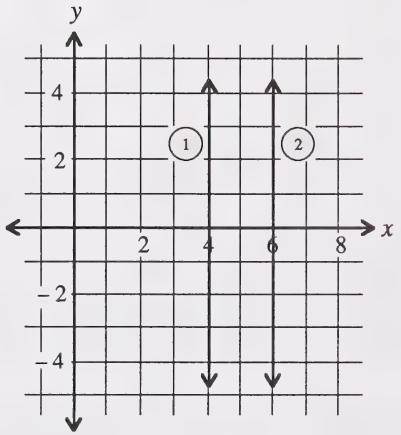
$2x = 16$ (1)
$x = 8$ (2),

neither equation can be put into the $y = mx + b$ form but both have the same x-intercept. In such cases, the system has an infinite number of solutions. Some of the solutions of the system are $(8, -2)$, $(8, 0)$, and $(8, 10)$.



When neither of the equations can be put into the $y = mx + b$ form and the x-intercepts of the equations are different, the system has no solutions. Such a system is

$x = 4$ (1)
$x = 6$ (2)



Try the following questions. In each question, determine whether the system has one solution, no solution, or an infinite number of solutions. Give reasons for your answers.

Do at least the odd-numbered questions.

12. $x+2y=14$ (1)
$y=2x+2$ (2)

13. $y=2x+5$ (1)
$x=y$ (2)

14. $x+2y=-10$ (1)
$x-4y=32$ (2)

15. $4x=10-2y$ (1)
$14x+7y=35$ (2)

16. $y=5x+4$ (1)
$y=5$ (2)

17. $y=3x+1$ (1)
$y=3x+2$ (2)

18. $x+2y=4$ (1)
$y=-\frac{1}{2}x+4$ (2)

19. $x=7$ (1)
$2x=11$ (2)

20. $x+5y=15$ (1)
$x=10$ (2)

For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.

Now that you can solve systems of equations, you can apply them to problem solving. The following steps are given to guide you to the answer in problem solving.

Step 1. Represent the unknown by variables.
Step 2. Represent the conditions in the problem by a system of equations.
Step 3. Solve the system of equations.
Step 4. Express your answer to the problem in sentence form and check your answer.

Example 4

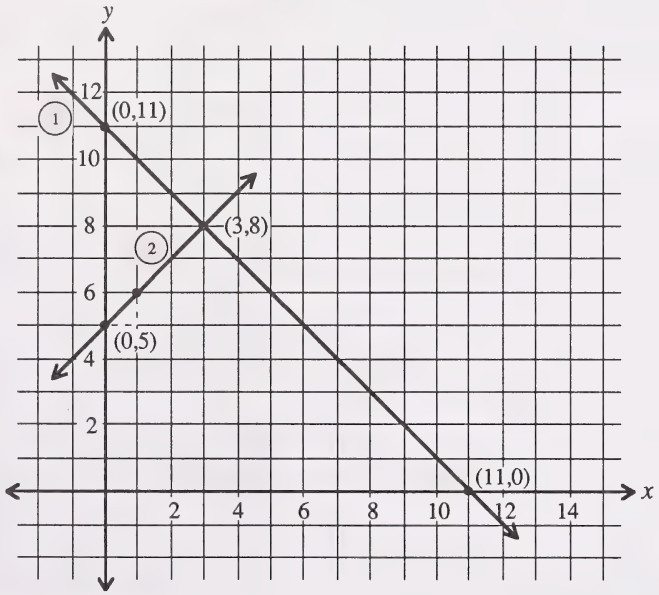
Which two numbers have a sum of 11 and a difference of 5?

Solution:

Step 1: Let x be the smaller of the two numbers and let y be the other number.

Step 2: Then,
$y+x=11$ (1)
and $y-x=5$ (2).

Step 3: For ①, the intercepts are (0, 11) and (11, 0).
In the slope-intercept form, ② becomes $y = x + 5$. The slope is 1 and the y-intercept is (0, 5).



The point of intersection is (3, 8).

The solution of the system of equations is $x = 3$ and $y = 8$.

Step 4:

The numbers are 8 and 3.

Check:

These numbers have a sum of 11 and a difference of 5 as required.

Example 5

Suppose that when Shauna counted her turkeys and goats, she found that she had seven animals. These animals had twenty legs. How many turkeys did she have and how many goats did she have?

Solution:

Step 1: Let the number of turkeys be x and the number of goats be y. Then the number of turkey legs will be $2x$ and the number of goat legs will be $4y$.

Step 2: Therefore,

$$2x + 4y = 20 \quad ①$$
$$\text{and } x + y = 7. \quad ②$$

Step 3: For ①: When $x = 0$,

$$2x + 4y = 20.$$
$$2(0) + 4y = 20$$
$$4y = 20$$
$$y = 5$$

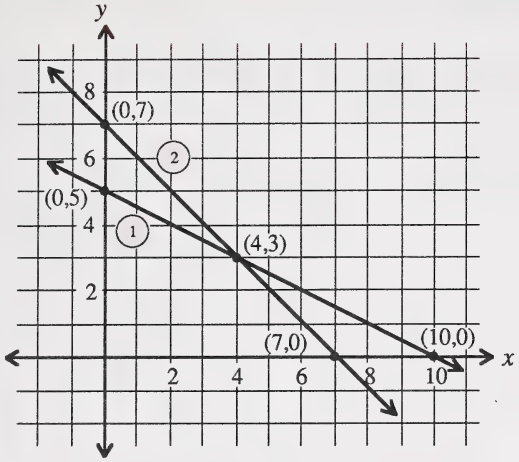
The y-intercept is (0, 5).

When $y = 0$,

$$2x + 4y = 20.$$
$$2x + 4(0) = 20$$
$$2x = 20$$
$$x = 10$$

The x-intercept is (10, 0).

For ②, the intercepts are (0, 7) and (7, 0).



The intersection of the graphs is (4, 3).

The solution of the system is $x = 4$ and $y = 3$.

Step 4: There are 4 turkeys and 3 goats.

Check:

There are 4(2) = 8 turkey legs and 3(4) = 12 goat legs for a total of 20 legs. There are 4 + 3 = 7 animals. The answer is the correct one for the problem.

Example 6

Ana and Joan bought golf balls at the pro-shop. The shop offered balls at two prices - one for old; another for new. Ana bought five old balls and two new balls for eighteen dollars. Joan bought four old balls and four new balls for twenty-four dollars. What were the prices for old and new balls?

Solution:

Let x dollars be the price for an old ball and
y dollars be the price for a new ball.

Then,

$5x + 2y = 18$ (1)
and $4x + 4y = 24$. (2)

In the $y = mx + b$ form, (1) becomes $y = -\frac{5}{2}x + 9$.

A short table of values for this equation is as follows.

x	y
0	9
4	−1
6	−6

For (2): When $x = 0$,

$$4x + 4y = 24.$$
$$4(0) + 4y = 24$$
$$4y = 24$$
$$y = 6$$

The y-intercept is (0, 6).

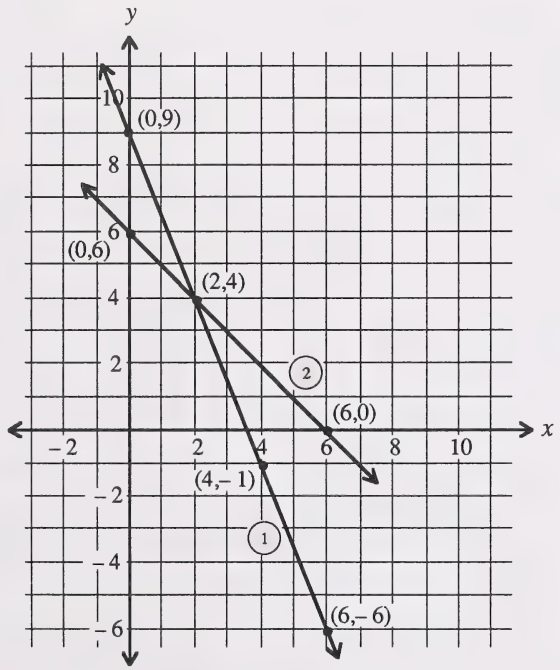
When $y = 0$,

$4x + 4y = 24.$

$4x + 4(0) = 24$

$4x = 24$

$x = 6$ The x-intercept is (6, 0).



The graphs cross at (2, 4).

The solution to the system of equations is $x = 2$ and $y = 4$.

The price of each old ball is two dollars and the price of each new ball is four dollars.

At these prices Joan would spend 4($2) + 4($4) = $24 and Ana would spend 5($2) + 2($4) = $18. This meets the conditions of the problem.

Try the following questions. Do at least two of them. If you need additional practice, do the rest of the questions. Turn to **Appendix B** for graph paper.

21. A rectangular garden has a length that exceeds the width by 4 m. The perimeter of the garden is 32 m. Give the dimensions of the garden.

22. A boy walks at a rate of 4 km every hour. His older sister walks at a rate of 5 km every hour. Suppose both decide to walk down a country road at their normal walking speeds. If the boy is allowed a head start of 2 km, how long will it take for his sister to catch up?

    Hint: You can use the formulas $y = 4x + 2$ and $y = 5x$ in solving the problem.

23. Four times a number minus three times another is 16. The sum of the numbers is 11. What are the numbers?

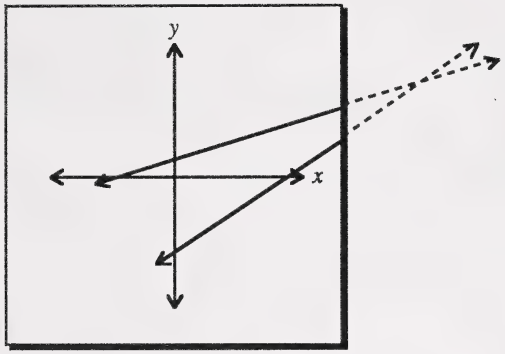
24. A certain money bag was known to contain two-dollar bills and five-dollar bills. The total value of the bills was $16.00. If there were five bills in the bag, how many of each kind of bill were there?

For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.

Activity 2

Solve systems of equations algebraically.

Solving systems graphically has its limitations. The graphs may intersect far away from the origin so it is hard to fit enough of the graph on the graph paper.



Unless the co-ordinates of the intersection are integers, they may be difficult to determine accurately. For example, it is not easy to see where the intersection point $\left(-\frac{7}{12}, \frac{3}{4}\right)$ would be on a graph. Besides all this, it does become tedious to keep drawing graphs. Fortunately, systems of equations can be solved without graphs; they can be solved algebraically.

One way to solve systems involves adding or subtracting equations in order to get rid of one of the variables. This is called the method of elimination and is shown for System A.

$$\text{A} \begin{cases} 2x-8=-y & (1) \\ 2x-y=4 & (2) \end{cases}$$

After the variables are placed in corresponding positions, the equations may be added in order to eliminate y. You can then solve for x.

$$\begin{array}{rrl} (1) \text{ rearranged:} & 2x+y=8 & (3) \\ & 2x-y=4 & (2) \\ \hline (2)+(3): & 4x+0=12 & \\ & 4x=12 & \\ & x=3 & \end{array}$$

Once you have the value for one variable, you can substitute it into either one of the original equations in order to determine the value for the other variable.

The value 3 can be substituted for x in (1). Always substitute in one of the original equations. This means to substitute into the equations you were given at the very beginning of the problem.

$$\begin{aligned} 2x-8&=-y \\ 2(3)-8&=-y \\ 6-8&=-y \\ -2&=-y \\ y&=2 \end{aligned}$$

The solution of System A is $x = 3$ and $y = 2$.

Sometimes, one of the equations must be multiplied by a constant before adding or subtracting. This is shown for System B.

$$\text{B}\begin{cases} 3x+4y=3 & (1) \\ 2x+y=27 & (2) \end{cases}$$

In order to eliminate y, multiply (2) by a factor of 4 and then subtract the equations. The resulting equation is solved for x.

$$\begin{aligned} & & 3x+4y=3 & \quad (1) \\ 4\times(2): & & 8x+4y=108 & \quad (3) \\ (1)-(3): & & -5x+0=-105 & \\ & & -5x=-105 & \\ & & x=21 & \end{aligned}$$

To obtain the value for y, the value of 21 can be substituted for x in (2).

$$\begin{aligned} 2x+y&=27 \\ 2(21)+y&=27 \\ 42+y&=27 \\ y&=27-42 \\ y&=-15 \end{aligned}$$

The solution of System B is $x = 21$ and $y = -15$.

There are times when both equations must be multiplied by constants in order to eliminate a variable. This can be shown for the previous system, System B. That way, you will also see that it does not matter which variable you eliminate; you obtain the same solution.

To eliminate the x, multiply (1) by 2 and (2) by 3 and then subtract the equations. The resulting equation is solved for y.

$$\begin{aligned} 2\times(1): & \quad 6x+8y=6 & \quad (3) \\ 3\times(2): & \quad 6x+3y=81 & \quad (4) \\ (3)-(4): & \quad 0+5y=-75 & \\ & \quad 5y=-75 & \\ & \quad y=-15 & \end{aligned}$$

When -15 is substituted for y in (2), the value for x is obtained.

$$\begin{aligned} 2x+y&=27 \\ 2x+(-15)&=27 \\ 2x&=27+15 \\ 2x&=42 \\ x&=21 \end{aligned}$$

The solution of System B is again found to be $x = 21$ and $y = -15$, as expected.

The constants you use to multiply the equations come from the coefficient of x. Equation 1 is multiplied by the number in front of x in (2). Equation 2 is multiplied by the number in front of x in (1).

Example 7

Solve the system algebraically and check your work.

$y = 2x + 4$ (1)
$x + y = 10$ (2)

Solution:

(1) rearranged: $-2x + y = 4$ (3)

$$\begin{array}{rl} & x + y = 10 \ (2) \\ (3) - (2): & -3x + 0 = -6 \\ & -3x = -6 \\ & x = 2 \end{array}$$

From (2): $x + y = 10$
$(2) + y = 10$
$y = 8$

The solution of the system is $x = 2$ and $y = 8$.

Check: $x = 2, y = 8$

LS	RS
y	$2x + 4$
8	$2(2) + 4$
	$4 + 4$
	8

LS = RS

LS	RS
$x + y$	10
$2 + 8$	
10	

LS = RS

Example 8

Solve the system algebraically. Check your work.

$5x + 2y = 4$ (1)
$3x - 7y = 27$ (2)

Solution:

$$\begin{array}{rl} 7 \times (1): & 35x + 14y = 28 \ (3) \\ 2 \times (2): & 6x - 14y = 54 \ (4) \\ (3) + (4): & 41x + 0 = 82 \\ & 41x = 82 \\ & x = 2 \end{array}$$

From (1): $5x + 2y = 4$
$5(2) + 2y = 4$
$10 + 2y = 4$
$2y = -6$
$y = -3$

The solution of the system is $x = 2$ and $y = -3$.

Check: $x = 2, y = 3$

LS	RS
$5x + 2y$	4
$5(2) + 2(-3)$	
$10 - 6$	
4	

LS = RS

LS	RS
$3x - 7y$	27
$3(2) - 7(-3)$	
$6 + 21$	
27	

LS = RS

Compare the solution for Example 7, to that of Example 1, Activity 1.

The factor 7 comes from the coefficient of y in (2).
The factor of 2 comes from the coefficient of y in (1).

Example 9

Solve the system algebraically. Check your work.

$6x+11y=45$ (1)
$9x+16y=66$ (2)

Solution:

$9\times$ (1): $54x+99y=405$ (3)
$6\times$ (2): $54x+96y=396$ (4)
(3) − (4): $0+3y=9$
$3y=9$
$y=3$

From (1): $6x+11y=45$
$6x+11(3)=45$
$6x+33=45$
$6x=45-33$
$6x=12$
$x=2$

The solution of the system is $x = 2$ and $y = 3$.

Check: $x = 2, y = 3$

LS	RS
$6x + 11y$	45
$6(2) + 11(3)$	
$12 + 33$	
45	

LS = RS

LS	RS
$9x + 16y$	66
$9(2) + 16(3)$	
$18 + 48$	
66	

LS = RS

In the solution to Example 9, the coefficients of x were both made to equal 54. This is simply the product of the original coefficients of x. In the example, there is a way to keep the numbers smaller. You aim to make 18 as the coefficients of x. The number 18 is the lowest common multiple (L.C.M.) of 6 and 9, which are the two original coefficients of x.

Now, to determine the constant needed to multiply (1), you divide the L.C.M. 18 by the original coefficient of x in (1):

$$\frac{18}{6}=3$$

To determine the constant needed to multiply (2), you divide the L.C.M. 18 by the original coefficient of x in (2).

$$\frac{18}{9}=2$$

Using these constants to multiply the equations, the system becomes

$3\times$ (1): $18x+33y=135$ (5)
$2\times$ (2): $18x+32y=132$ (6)

When you subtract these equations you obtain an equation in y.

$18x+33y=135$ (5)
$18x+32y=132$ (6)
(5) − (6): $0+y=3$
$y=3$

Now you can determine that $x = 2$ in the same way as before.

Maybe you're impressed how using the L.C.M. keeps the calculations simpler. Look at the following example in which the L.C.M. of coefficients is used.

Example 10

Solve the system algebraically.
Check your work.

$7x+12y=142$ (1)
$31x-18y=202$ (2)

Solution:

See the margin box for the L.C.M.

The constant used to multiply (1) is $\frac{36}{12}=3$.

The constant used to multiply (2) is $\frac{36}{18}=2$.

$3\times$ (1): $21x+36y=426$ (3)
$2\times$ (2): $62x-36y=404$ (4)
(3) + (4): $83x+0=830$
$83x=830$
$x=\frac{830}{83}$
$x=10$

From (1): $7x+12y=142$
$7(10)+12y=142$
$70+12y=142$
$12y=72$
$y=6$

The solution of the system is $x = 10$ and $y = 6$.

LS	RS
$7x+12y$	142
$7(10)+12(6)$	
$70+72$	
142	

LS = RS

LS	RS
$31x-18y$	202
$31(10)-18(6)$	
$310-108$	
202	

LS = RS

The L.C.M. of the coefficients of the variable to be eliminated is used.

$12=3\times2\times2$
$18=3\times3\times2$
L.C.M. $=3\times3\times2\times2$
$=36$

The L.C.M. of the coefficients of y were used. It is y that disappears.

Try the following. Do at least the even-numbered questions. Do the rest of the questions if you need additional practice. Solve each system of equations algebraically using the method of elimination. Check your solutions.

1. $x = -y$ (1)
   $y = x + 3$ (2)

   (Hint: Rearrange the variables.)

2. $2y - 3x = 6$ (1)
   $3x - y = 12$ (2)

3. $5x + 2y = 18$ (1)
   $4x + 4y = 24$ (2)

4. $3x + 7y = 46$ (1)
   $5x - 9y = -6$ (2)

5. $16x + 9y = 15$ (1)
   $6y - 4x = -1$ (2)

6. $40x + 31y = 692$ (1)
   $60x - 17y = 276$ (2)

7. $32 - 2x = 5y$ (1)
   $3y - 5x = -18$ (2)

8. $10a - 3b = 4$ (1)
   $21b - 12a = 1$ (2)

   (Solving this system means giving the values of each of a and b so that the equations are true.)

9. $13x + 36y = -259$ (1)
   $17x - 24y = 301$ (2)

10. $5m - 3n = 5$ (1)
    $2m + 9n = 2$ (2)

    (Solving this system means giving the values of m and n so that the equations are true.)

11. $2y = x - 5$ (1)
    $2x = y + 3$ (2)

For solutions to **Activity 2**, turn to **Appendix A, Topic 1**.

There is another method that can be used to solve equations algebraically: the method of substitution. This is shown for System C.

$$C\begin{cases}2x+3y=10 & (1)\\ y=4x-34 & (2)\end{cases}$$

From (2), an equivalent expression for y in terms of x is obtained. This expression is substituted for y in (1).

$$2x+3(4x-34)=10, \text{ since } y=4x-34$$

This can be used to obtain the value for x.

$$\begin{aligned}2x+12x-102&=10\\ 14x&=10+102\\ 14x&=112\\ x&=8\end{aligned}$$

The value of 8 can be substituted for x in one of the original equations in order to obtain a value for y.

$$\begin{aligned}\text{From } (2):\ y&=4(8)-34\\ y&=32-34\\ y&=-2\end{aligned}$$

The solution of System C is $x = 8$ and $y = -2$.

When using the method of substitution, you can substitute for either variable; however, when possible, you should pick the variable that can be expressed in terms of the other without the introduction of fractions. In System C, x can be expressed as either

$$x=-\frac{3}{2}y+5 \text{ as obtained from (1) or}$$

$$x=\frac{1}{4}y+8\frac{1}{2} \text{ as obtained from (2).}$$

This gives you an equation that can be used to give the value for y.

$$2\left(\frac{1}{4}y+8\frac{1}{2}\right)+3y=10 \text{ when } x \text{ is replaced in (1),}$$

or

$$y=4\left(-\frac{3}{2}y+5\right)-34 \text{ when } x \text{ is replaced in (2).}$$

Each of these equations may be solved to determine that $y = -2$, but you wouldn't want to. These equations are awkward to work with.

Look at the following examples.

Both equations must be used: one equation is used to solve for an equivalent expression; substitution with this expression takes place in the other equation.

Example 11

Solve the system using the method of substitution. Check your solution.

$2x+3y=-10$ (1)

$x+5y=-33$ (2)

Solution:

From (2), $x=-5y-33$.

Substituting for x in (1) and solving for y

$$\begin{aligned} 2(-5y-33)+3y&=-10\\ -10y-66+3y&=-10\\ -66-7y&=-10\\ -7y&=-10+66\\ -7y&=56\\ y&=-8 \end{aligned}$$

From (2): $x+5(-8)=-33$

$$\begin{aligned} x-40&=-33\\ x&=40-33\\ x&=7 \end{aligned}$$

The solution of the system is $x=7$ and $y=-8$.

LS	RS
$2x+3y$	-10
$2(7)+3(-8)$	
$14+(-24)$	
-10	

LS = RS

LS	RS
$x+5y$	-33
$7+5(-8)$	
$7+(-40)$	
-33	

LS = RS

Example 12

Let the temperature in Fahrenheit be represented by F and let the temperature in Celsius be C. Then the relation between these scales is expressed by

$$C=\frac{5}{9}(F-32).$$

Determine the temperature at which the scales have the same reading by solving the system.

$C=\frac{5}{9}(F-32)$ (1)

$C=F$ (2)

Solution:

Using ② to substitute for F in ①: $C = \frac{5}{9}(C - 32)$

Solve for C. $9C = 5(C - 32)$

$9C = 5C - 160$

$4C = -160$

$C = -40$

From ②, $-40 = F$

$F = -40.$

The solution of the system is $C = -40$ and $F = -40$.

LS	RS
C	$\frac{5}{9}(F - 32)$
(-40)	$\frac{5}{9}[(-40) - 32]$
	$\frac{5}{9}(-72)$
	-40

LS = RS

LS	RS
C	F
(-40)	(-40)

LS = RS

The Fahrenheit and Celsius scales have the same reading at – 40°C (or – 40°F).

Example 13

Solve the system using the method of substitution. Check your solution.

$x - y = -8$ ①

$y = \frac{3}{2}x + 4$ ②

Solution:

From ①, $x = y - 8$.
Substitute for x in ② and solve for y.

$y = \frac{3}{2}(y - 8) + 4$

$2y = 3(y - 8) + 8$

$2y = 3y - 24 + 8$

$-y = -16$

$y = 16$

From ①, $x - 16 = -8$,

$x = 8.$

The solution is $x = 8$ and $y = 16$.

Check: $x = 8, y = 16$

LS	RS
$x - y$	-8
$(8) - (16)$	
-8	

LS = RS

LS	RS
y	$\frac{3}{2}x + 4$
(16)	$\frac{3}{2}(8) + 4$
16	$12 + 4$
	16

LS = RS

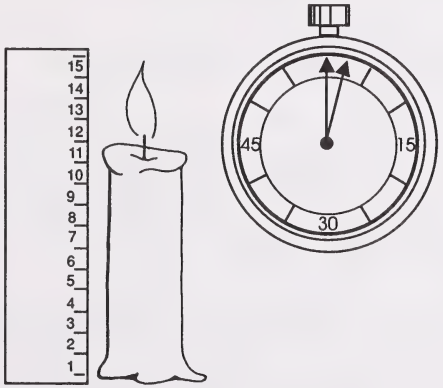
Now that you have seen several systems solved by substitution, you're ready to use this method on your own.

Solve the following systems using the method of substitution. Check your solutions. Do 12 and at least three other questions.

12. Suppose the height of a burning candle can be determined by the equation.

$h = 15 - 2t,$

where h cm is the candle height and
t hours is the length of time the candle has been lit.



The system $h = 15 - 2t$ (1)
$h = t$ (2)

can be solved to determine when the same number gives the candle height and the length of time the candle has been burning.

Solve the system using substitution, check your solution, and tell when the same number gives the height and the time.

13. $4x - 16y = 3$ (1)
$x - 2y = 0$ (2)

14. $5m + 2n = 44$ (1)
$3m + n = 24$ (2)

15. $20p + 6q = 3$ (1)
$2p = -q$ (2)

16. $10x + 2y = -1$ (1)
$2x = 1 - y$ (2)

17. $x + y = 10$ (1)
$x - y = 10$ (2)

For solutions to **Activity 2**, turn to **Appendix A, Topic 1**.

As you found with graphing, there are systems that have no solutions and still others with an infinite number of solutions. How can you tell which is the case when you are trying to solve systems algebraically? This happens when the algebraic approach leads to an equation with no variables. If there is no solution, the resulting expression will be false. If there is an infinite number of solutions, the resulting equation will be true.

D $\begin{cases} 2y-3x=6 & (1) \\ 3x-2y=12 & (2) \end{cases}$

You can see how an algebraic approach shows that System D has no solutions.

From (1) with variables rearranged:

$$\begin{array}{rr} & -3x+2y=6 \quad (3) \\ & 3x-2y=12 \quad (2) \\ \hline (3)+(2): & 0=18 \end{array}$$ which is false.

This shows that System D has no solutions.

E $\begin{cases} y=-2x-4 & (1) \\ 4x+2y=-8 & (2) \end{cases}$

For System E, you can see that an algebraic approach shows that the system has an infinite number of solutions.

Substituting for y in (2) and solving for x,

$$\begin{aligned} 4x+2(-2x-4) &= -8 \\ 4x-4x-8 &= -8 \\ -8 &= -8 \text{ which is true.} \end{aligned}$$

Try the following questions.

For each system, use the method of elimination or method of substitution to determine whether the system has one solution, no solution, or an infinite number of solutions.

18. $x+\frac{4}{5}y=2$ (1)
$5x+4y=10$ (2)

19. $3y-2x=5$ (1)
$3y-2x=1$ (2)

For solutions to **Activity 2**, turn to **Appendix A, Topic 1**.

You can easily see how many solutions a system has when the equations are expressed in the $y = mx + b$ form. However, you now know why things may seem to go wrong when looking for a solution algebraically: the system may have no solution or an infinite number of solutions.

When you could only solve systems graphically you could only apply them to problems in which the unknowns were small integers. Now, solving systems algebraically, you'll be able to apply systems of equations to a greater variety of problems.

Look at the examples in which systems of equations are applied.

In question 7 of Activity 1, you used a graph to determine that this system has no solutions.

In question 8 of Activity 1, you used a graph to determine that this system has an infinite number of solutions.

Remember the problem-solving steps given in Activity 1.

Step 1. Represent the unknowns by variables.
Step 2. Represent the conditions of the problem by a system of equations.
Step 3. Solve the system of equations.
Step 4. Express your answer to the problem in sentence form and check your answer.

Example 14

Jimmy looked into Farmer Joe's pen and saw bunnies and chicks. He counted 65 heads and 210 feet. How many bunnies and how many chicks were in the pen?

Solution:

Step 1: Let the number of bunnies be x and the number of chicks be y. Then, the number of bunny feet will be $4x$ and the number of chick feet will be $2y$.

Step 2: Therefore,

$$4x+2y=210 \quad (1)$$
$$x+y=65. \quad (2)$$

Step 3: $4\times(2)$: $4x+4y=260$ (3)

$$4x+2y=210 \quad (1)$$

(3) − (1): $0+2y=50$

$$2y=50$$
$$y=25$$

From (1): $x+y=65$

$$x+25=65$$
$$x=40.$$

Step 4: The solution of the system is $x = 40$ and $y = 25$.

There were 40 bunnies and 25 chicks.

There were 40 + 25 = 65 heads and
40(4 feet) + 25(2 feet) = (160 + 50) feet = 210 feet. This shows that the answer meets the conditions in the problem.

Example 15

I. M. Strong decided to buy some new gym wear. He spent $126.00 on shorts and T-shirts. If one pair of shorts cost $15.00 and one T-shirt cost $12.00, how many of each did he buy if he bought 9 items in total?

Solution:

Let s represent the number of shorts and t the number of T-shirts.

Then the amount spent on shorts would be $15s$ dollars and the amount spent on T-shirts would be $12t$ dollars.

$$15s+12t=126 \quad (1)$$
$$s+t=9 \quad (2)$$

$$15s+12t=126 \quad (1)$$

$12\times(2)$: $12s+12t=108$ (3)

(1) − (3): $3s=18$

$$s=6$$

From (1): $s+t=9$

$$6+t=9$$
$$t=9-6$$
$$t=3$$

The solution of the system is $s = 6$ and $t = 3$.
I. M. Strong would have bought 6 shorts and 3 T-shirts.

His purchase would have cost 6($15) + 3($12) = $90 + $36 = $126. He would have bought 6 items + 3 items = 9 items. This corresponds with the conditions in the problem.

Example 16

The sum of Karen's age and her mother's age is 49. Four years ago, twice the mother's age plus three times Karen's age was 92. What are their present ages?

Solution:

Let Karen's age be x and her mother's age be y.
Four years ago their ages would have been

$$x+y=49 \quad (1)$$
$$3(x-4)+2(y-4)=92 \quad (2)$$

Simplifying (2): $3x-12+2y-8=92$

$$3x+2y-20=92$$
$$3x+2y=112 \quad (3)$$

$3\times(1)$: $3x+3y=147$ (4)

$3x+2y=112$ (3)

(4) − (3): $y=35$

From (1): $x+y=49$

$x+35=49.$

$x=14$

The solution of the system is $x = 14$ and $y = 35$.

Presently, Karen's age is 14 and her mother's age is 35.

Their present ages add up to 49.
Four years ago, Karen's age was $14 - 4 = 10$ and her mother's age was $35 - 4 = 31$. Three times 10 plus two times 31 is $30 + 62 = 92$. Checking the answer shows it to be correct.

Example 17

A farmer built a rectangular pig pen so that its length is three times its width. The pen was enclosed by 640 m of fence. What are the dimensions of the pen?

Solution:

Let the width be x metres and the length be y metres.
Then

$$(1) \quad y=3x$$
$$(2) \quad 2x+2y=640$$

Substitute for y in (2) and solve for x.

$$2x+2(3x)=640$$
$$2x+6x=640$$
$$8x=640$$
$$x=80$$

$P = 2l + 2w$

From (1), $y = 3(80)$

$y = 240$

The solution of the system is $x = 80$ and $y = 240$.

The pen is 80 m wide and 240 m long.

For such a pen, the length is three times the width.
The perimeter can be calculated as follows:

$$\begin{aligned} P &= 2l + 2w \\ &= 2(240\text{ m}) + 2(80\text{ m}) \\ &= 480\text{ m} + 160\text{ m} \\ &= 640\text{ m} \end{aligned}$$

This shows that the length and width meet the requirements of the problem.

Try the following questions. Do at least four of them. If you need additional practice, do the rest of the questions.

20. Two numbers have a sum of 100. Their difference is 124. What are the numbers? (The problem does not require both numbers to be positive.)

21. Three times one number and five times another add up to 595. The sum of the numbers themselves is 165. What are these numbers?

22. Bill Hroshee has 46 coins in his pocket. All the coins are either quarters or dimes. The value of these coins is \$6.25. How many coins of each kind does he have?

23. Suppose the sum of two numbers is 601 and their difference is 89. What are the numbers?

24. In a darts contest, a bull's-eye counts as 10, and the outer zone counts as 6. Determine how many of each of these a contestant makes in scoring 62 with a total of 9 shots.

25. Frances cashed a \$350 cheque at the bank. The teller gave Frances the amount in ten- and twenty-dollar bills and she received a total of nineteen bills. How many of each kind of bill did Frances receive?

26. The sum of two numbers is 54. Five times one of these numbers exceeds twice the other number by 109. What are the numbers?

27. Jan bought sandwiches for a dozen people at a corner store. At first, she wanted seven ham sandwiches and five cheese sandwiches. This would have cost her \$21.50. But two people changed their minds so she ended up buying five ham sandwiches and seven cheese sandwiches. For this, she was charged \$20.50. Determine the price for each type of sandwich.

For solutions to **Activity 2**, turn to **Appendix A, Topic 1**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

One way to solve a system of equations is to graph each of the equations. The coordinates of the intersection point give the solution.

Example 18

Solve the system of equations and check your work.

$5x+2y=20$ (1)

$y=\frac{1}{2}x+4$ (2)

Solution:

For (1): When $x=0$,

$5x+2y=20$

$5(0)+2y=20$

$2y=20$

$y=10$

The y-intercept is (0, 10).

When $y=0$,

$5x+2y=20$

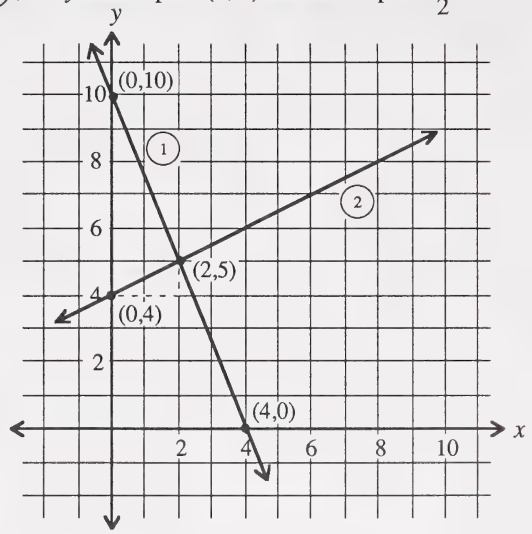
$5x+2(0)=20$

$5x=20$

$x=4$

The x-intercept is (4, 0).

For (2), the y-intercept is (0, 4) and the slope is $\frac{1}{2}$.



The intersection of the graphs is (2, 5). The solution of the system is $x=2$ and $y=5$.

Check: $x = 2, y = 5$

LS	RS
$5x + 2y$	20
$5(2) + 2(5)$	
$10 + 10$	
20	

LS = RS

LS	RS
y	$\frac{1}{2}(2)+4$
5	$1+4$
	5

LS = RS

You can determine the number of solutions of a system by comparing the equations in the $y = mx + b$ form.

If the slopes are equal and the y-intercepts are unequal, the system has no solution.

If the slopes are equal and the y-intercepts are equal, the system has an infinite number of solutions.

Example 19

Determine the number of solutions of the given system.

$$5x+2y=20 \quad (1)$$

$$y=\frac{1}{2}x+4 \quad (2)$$

Solution:

$$5x+2y=20$$

$$2y=-5x+20$$

$$y=-\frac{5}{2}x+10$$

The slope for (1) is $-\frac{5}{2}$, and for (2) it is $\frac{1}{2}$. Since the slopes are unequal, the system has one solution.

Example 20

How many solutions does the system have?

$$5x-y=2 \quad (1)$$

$$15x-3y=7 \quad (2)$$

Solution:

$$5x-y=2 \quad (1)$$

$$-y=-5x+2$$

$$y=5x-2$$

$$15x-3y=7 \quad (2)$$

$$-3y=-15x+7$$

$$y=5x-\frac{7}{3}$$

The slopes for both equations are equal. For (1), the y-intercept is -2, and for (2), the y-intercept is $-\frac{7}{3}$. The y-intercepts are unequal. It follows that the system has no solutions.

Systems may also be solved algebraically. Equations in which the coefficients are the same size are added or subtracted in order to obtain an equation in one variable. The resulting equation is solved for one of the variables. The value for this variable is substituted into one of the equations of the system so that the value for the other variable can be obtained.

Example 21

Solve the system algebraically and check your work.

$x+2y=11$ (1)

$5x-2y=19$ (2)

Solution:

$x+2y=11$ (1)

$5x-2y=19$ (2)

(1) + (2): $6x+0=30$

$6x=30$

$x=5$

From (1): $x+2y=11$

$(5)+2y=11$

$2y=6$

$y=3$

The solution of the system is $x=5$ and $y=3$.

Check: $x=5, y=3$

LS	RS
$x+2y$	11
$5+2(3)$	
$5+6$	
11	

LS = RS

LS	RS
$5x-2y$	19
$5(5)-2(3)$	
$25-6$	
19	

LS = RS

Example 22

Solve the system algebraically and check your work.

$7x-5y=-27$ (1)

$8x+3y=4$ (2)

Solution:

$3\times$(1): $21x-15y=-81$ (3)

$5\times$(2): $40x+15y=20$ (4)

(3) + (4): $61x=-61$

$x=-1$

From (2): $8x+3y=4$

$8(-1)+3y=4$

$-8+3y=4$

$3y=12$

$y=4$

The solution of the system is $x=-1$ and $y=4$.

Check: $x=-1, y=4$

LS	RS
$7x-5y$	-27
$7(-1)-5(4)$	
$-7-20$	
-27	

LS = RS

LS	RS
$8x+3y$	4
$8(-1)+3(4)$	
$-8+12$	
4	

LS = RS

The equations of the system are multiplied by suitable constants so that for one of the variables, the coefficients become the same size.

Try the following questions. Turn to **Appendix B** for graph paper.

1. Solve the following system of equations graphically.

$x+3y=6$ (1)
$x+y=4$ (2)

2. Determine the number of solutions of the system below.

$21x-3y=1$ (1)
$3y-21x=1$ (2)

3. Determine the number of solutions of the following system.

$31x-y=-10$ (1)
$17x+y=8$ (2)

4. Solve the following system algebraically. Check your work.

$2x-3y=0$ (1)
$5x+3y=21$ (2)

5. Solve the following system algebraically. Check your work.

$3x+4y=5$ (1)
$2x+7y=12$ (2)

For solutions to the **Extra Help**, turn to **Appendix A, Topic 1**.

Extensions

Systems having three equations and three unknowns can also be solved by a method of elimination: equations are combined in such a way that equations with fewer variables are obtained. In this way, an equation in one variable can be produced.

See the examples.

Example 23

Solve the following system and check your solution.

$x+y=5$ (1)
$x+z=7$ (2)
$y+z=6$ (3)

Solution:

$$\begin{array}{rr} & x+y+0z=5 \quad (1) \\ & x+0y+z=7 \quad (2) \\ (1)-(2): & \overline{\qquad y-z=-2 \quad (4)} \end{array}$$

$$\begin{array}{rr} & y+z=6 \quad (3) \\ & y-z=-2 \quad (4) \\ (3)+(4): & \overline{2y \quad =4} \\ & y=2 \end{array}$$

From ①: $x+y=5$

$x+(2)=5$

$x=3$

From ③: $y+z=6$

$(2)+z=6$

$z=4$

The solution is $x=3$, $y=2$, and $z=4$.

Check: $x=3$, $y=2$, $z=4$

LS	RS
$x+y$	5
(3) + (2)	
5	

LS = RS

LS	RS
$x+z$	7
(3) + (4)	
7	

LS = RS

LS	RS
$y+z$	6
(2) + (4)	
6	

LS = RS

Example 24

Solve the following system and check your solution.

$x+2y+3z=13$ ①

$4x+3y+4z=23$ ②

$3x+5y+5z=26$ ③

Solution:

$4 \times$ ①: $4x+8y+12z=52$ ④

$4x+3y+4z=23$ ②

④ − ②: $5y+8z=29$ ⑤

$4 \times$ ③: $12x+20y+20z=104$ ⑥

$3 \times$ ②: $12x+9y+12z=69$ ⑦

⑥ − ⑦: $11y+8z=35$ ⑧

$11y+8z=35$ ⑧

$5y+8z=29$ ⑤

⑧ − ⑤: $6y+0=6$

$6y=6$

$y=1$

From ⑧: $11y+8z=35$

$11(1)+8z=35$

$11+8z=35$

$8z=24$

$z=3$

From ①: $x+2y+3z=13$

$x+2(1)+3(3)=13$

$x+2+9=13$

$x+11=13$

$x=2$

The solution of the system is $x=2$, $y=1$, and $z=3$.

Check: $x = 2, y = 1, z = 3$

LS	RS
$x + 2y + 3z$	13
$(2)+2(1)+3(3)$	
$2 + 2 + 9$	
13	

LS = RS

LS	RS
$4x + 3y + 4z$	23
$4(2)+3(1)+4(3)$	
$8 + 3 + 12$	
23	

LS = RS

LS	RS
$3x + 5y + 5z$	26
$3(2)+5(1)+5(3)$	
$6 + 5 + 15$	
26	

LS = RS

Try the following questions.

Solve the following systems of equations and check your solutions.

1. $a+b=2$ (1)
   $a+c=7$ (2)
   $b+c=-1$ (3)

2. $2x+2y+z=3$ (1)
   $x+y=1$ (2)
   $3x+2y=0$ (3)

3. $x+y+z=-3$ (1)
   $2x+3y-z=-13$ (2)
   $4x-5y+3z=-5$ (3)

For solutions to **Extensions**, turn to **Appendix A, Topic 1**.

Unit Summary

What You Have Learned

Not only have you learned the meaning of solving a system of equations, but you learned several techniques for solving them: systems may be solved graphically or algebraically. You also found that systems may have exactly one solution, no solution, or an infinite number of solutions. Furthermore, you saw that systems of equations are powerful tools in a variety of applications.

You are now ready to

complete the **Unit Assignment**.

Appendices

Appendix A
Solutions

Review

Topic 1 Solving Systems of Equations

Appendix B
Graph Paper

Appendix A Solutions

Review

1. $5x-10+2x=60$

$7x-10=60$

$7x=70$

$x=10$

2. $2x-3y=-12$

$-3y=-2x-12$

$3y=2x+12$

$y=\frac{2}{3}x+4$

3. Let the number of horses be n.
Then the number of legs is $4n$.

$4n-n=21$

$3n=21$

$n=7$

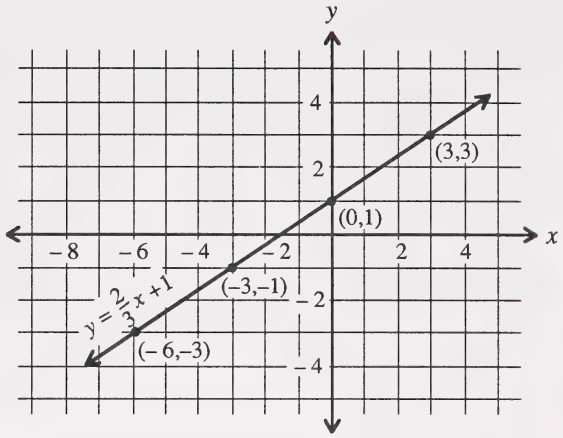
There are seven horses.
Check: Seven horses have 28 legs. There are 21 more legs than heads, as there are supposed to be.

4. For A, $x=-4$ and $y=-5$. A is $(-4,-5)$.
For B, $x=4$ and $y=6$. B is $(4,6)$.

5. $y=\frac{2}{3}x+1$

x	y
-6	-3
-3	-1
0	1
3	3

Points on the graph are $(-6,-3)$, $(-3,-1)$, $(0,1)$, and $(3,3)$.



6. $y=\frac{4}{5}x+\frac{3}{5}$

When $x=3$,

$y=\frac{4}{5}(3)+\frac{3}{5}$

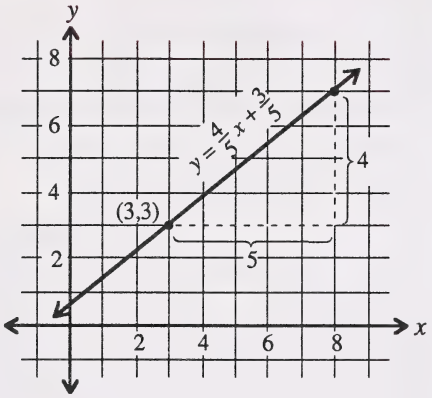
$=\frac{12}{5}+\frac{3}{5}$

$=\frac{15}{5}$

$=3.$

One point of the graph is (3, 3).

The slope is $\frac{4}{5}$.



7. $8x+5y=40$

When $y=0$,

$8x+5(0)=40.$

$8x=40$

$x=5$

The x-intercept is (5, 0).

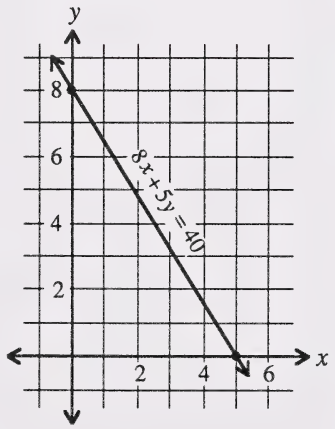
When $x=0$,

$8(0)+5y=40.$

$5y=40$

$y=8$

The y-intercept is (0, 8).



Exploring Topic 1

Activity 1

Solve systems of equations graphically.

1. $x+y=7$ (1)

$x-y=1$ (2)

For (1): $x+y=7$

When $y=0$,

$x+(0)=7.$

$x=7$

The x-intercept is (7, 0).

When $x=0$,

$(0)+y=7.$

$y=7$

The y-intercept is (0, 7).

For (1): $x-y=1$

When $y=0$,

$x-(0)=1.$

$x=1$

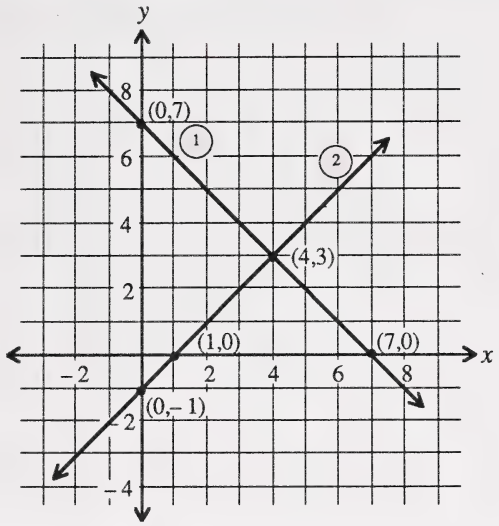
The x-intercept is (1, 0).

When $x = 0$,

$(0) - y = 1.$

$y = -1$

The y-intercept is $(0, -1)$.



The intersection of the graphs is (4, 3).

The solution of the system is $x = 4$ and $y = 3$. This solution can also be expressed as (4, 3).

Check: $x = 4, y = 3$

LS	RS
$x + y$	7
$4 + 3$	
7	

LS = RS

LS	RS
$x - y$	1
$4 - 3$	
1	

LS = RS

2. $x - 3y = -6$ (1)

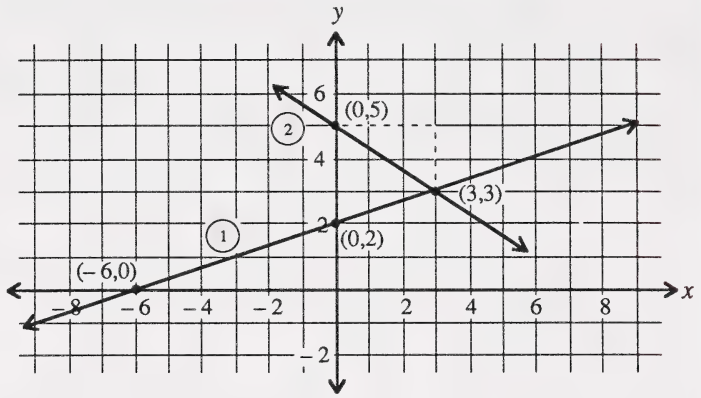
$2x + 3y = 15$ (2)

For (1), the intercepts are (0, 2) and (− 6, 0).

For (2): $3y = -2x + 15$

$y = -\frac{2}{3}x + 5$

The slope is $-\frac{2}{3}$ and the y-intercept is (0, 5).



The intersection of the graphs is (3, 3). The solution of the system is $x = 3$ and $y = 3$ or (3, 3).

Check: $x = 3, y = 3$

LS	RS
$x - 3y$	-6
$-3(3)$	
$3 - 9$	
-6	

LS = RS

LS	RS
$2x + 3y$	15
$2(3) - 3(3)$	
$6 + 9$	
15	

LS = RS

3. $x - 4y = -20$ (1)
   $2x + y = -4$ (2)

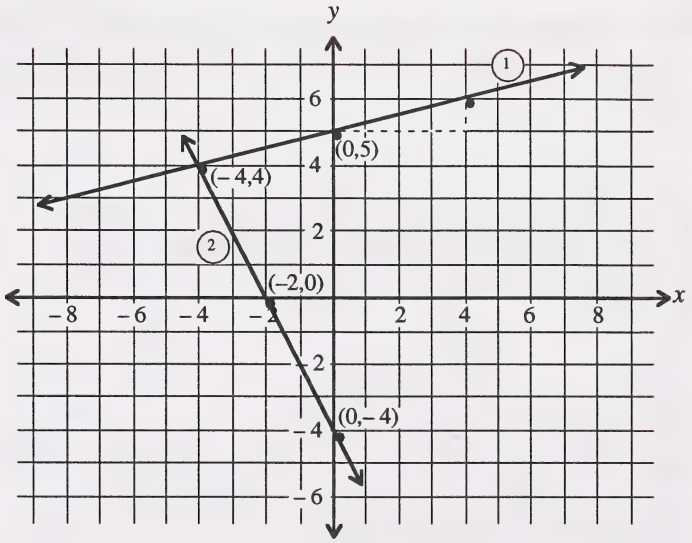
In the slope-intercept form, (1) becomes

$-4y = -x - 20$

$y = \frac{1}{4}x + 5$

The slope is $\frac{1}{4}$ and the y-intercept is (0, 5).

For (2), the intercepts are (– 2, 0) and (0, – 4).



The graphs intersect at (– 4, 4).

The solution of the system is $x = -4$ and $y = 4$. The solution can also be expressed as (– 4, 4).

Check: $x = -4, y = 4$

LS	RS
$x - 4y$	-20
$(-4) - 4(4)$	
$-4 - 16$	
-20	

LS = RS

LS	RS
$2x + y$	-4
$2(4) + 4$	
$-8 + 4$	
-4	

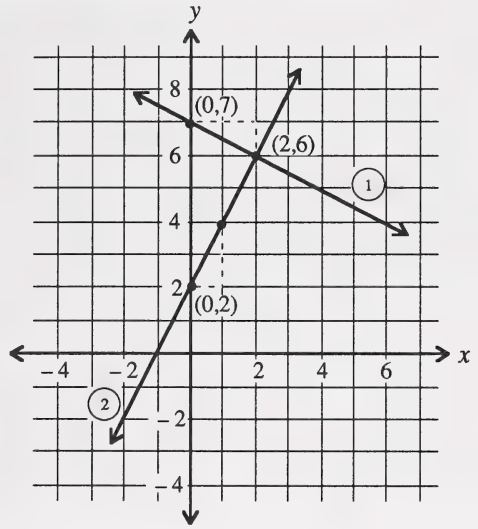
LS = RS

4. $x+2y=14$ (1)

$y=2x+2$ (2)

The slope of (2) is 2 and the y-intercept is (0, 2).

In the slope-intercept form (1) becomes $y=-\frac{1}{2}x+7$.
The slope is $-\frac{1}{2}$ and the y-intercept is (0, 7).



The graphs intersect at (2, 6).

The solution of the system is $x = 2$ and $y = 6$.

$x + 2y = 14$ (1)

LS	RS
$2 + 2(6)$	14
$2 + 12$	
14	

LS = RS

$y = 2x + 2$ (2)

LS	RS
6	$2(2) + 2$
	$4 + 2$
	6

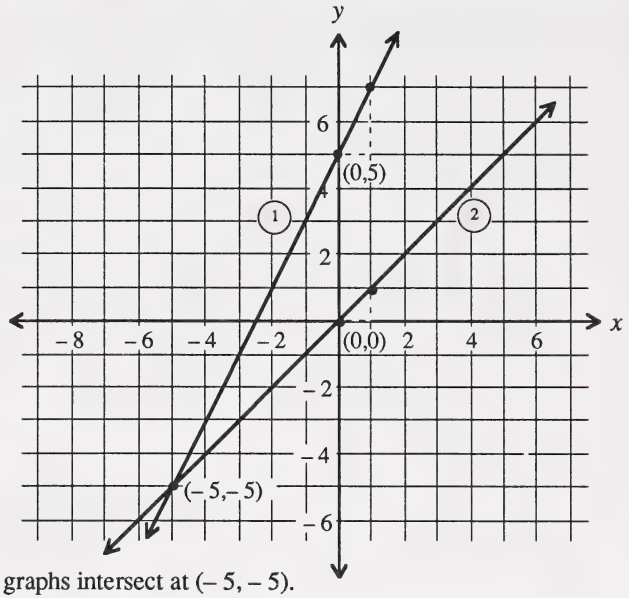
LS = RS

5. $y=2x+5$ (1)

$x=y$ (2)

For (1), the slope is 2 and the y-intercept is (0, 5).

The graph of (2) is a line through the origin and with a slope of 1.



The graphs intersect at (– 5, – 5).
The solution of the system is $x = -5$ and $y = -5$.
Check: $x = -5$, $y = -5$

LS	RS
y	$2x + 5$
-5	$2(-5) + 5$
	$-10 + 5$
	-5

LS = RS

LS	RS
x	y
-5	-5

LS = RS

6. $x+2y=-10$ (1)

$x-4y=32$ (2)

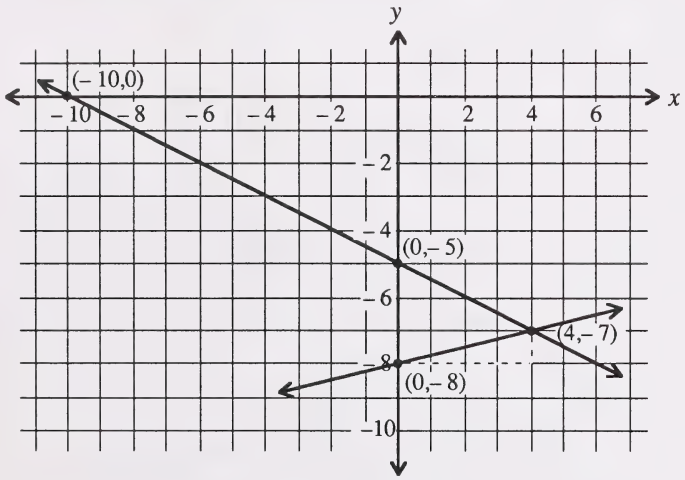
The intercepts of (1) are (0, – 5) and (–10, 0).

In the slope-intercept form, (2) becomes

$-4y=-x+32$

$y=\frac{1}{4}x-8$

The slope is $\frac{1}{4}$ and the y-intercept is (0, – 8).



The graphs intersect at (4, – 7).

The solution to the system is $x = 4$ and $y = – 7$.

Check: $x = 4$, $y = – 7$

LS	RS
$x + 2y$	– 10
$4 + 2(– 7)$	
$4 + (– 14)$	
– 10	

LS = RS

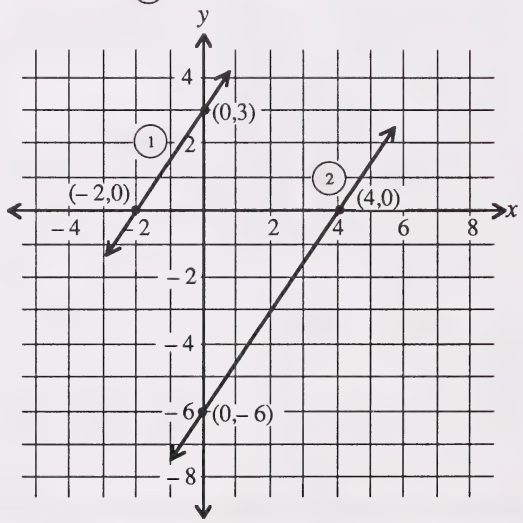
LS	RS
$x – 4y$	32
$4 – 4(– 7)$	
$4 – (– 28)$	
32	

LS = RS

7. $2y-3x=6$ (1)

$3x-2y=12$ (2)

The intercepts of (1) are (0, 3) and (– 2, 0).
The intercepts of (2) are (0, – 6) and (4, 0).

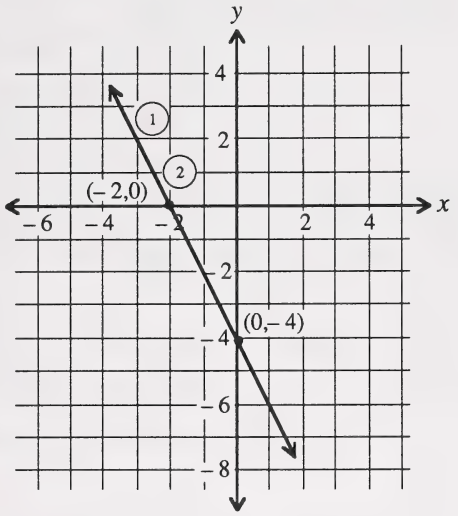


The lines have the same slope and are parallel. They do not intersect. Graphing yields no solution.

8. $y=-2x-4$ (1)
$4x+2y=-8$ (2)

For (1), the slope is – 2 and the y-intercept is (0, – 4).

For (2), the intercepts are (0, – 4) and (– 2, 0).



The graphs coincide.

Any solution of either equation is a solution of the system.
All points on the lines satisfy the system.

9. $x-4y=-20$ (1)
$2x+y=-4$ (2)

In the slope-intercept form, (1) is $y=\frac{1}{4}x+5$.

In the slope-intercept form, (2) is $y=-2x-4$.

$m_1=\frac{1}{4}$ and $m_2=-2$

because $m_1 \neq m_2$, the system has exactly one solution.

You determined the solution in question 3.

10. $2y-3x=6$ (1)
$3x-2y=12$ (2)

In the slope-intercept forms, (1) is $y=\frac{3}{2}x+3$

and (2) is $y=\frac{3}{2}x-6$.

The slopes are equal. $m_1=m_2=\frac{3}{2}$

The y-intercepts are unequal. $b_1=3$ and $b_2=-6$

It follows that the system has no solutions. You discovered this in question 7.

11. $y=-2x-4$ (1)
$4x+2y=-8$ (2)

In the y-intercept form, (2) is $y=-2x-4$.

For this system, $m_1=m_2=-2$ and $b_1=b_2=-4$. Therefore, the system has an infinite number of solutions. You found this in question 8.

12. $x+2y=14$ (1)
$y=2x+2$ (2)

Placed in the y-intercept form, (1) becomes $y=-\frac{1}{2}x+7$.

In this system, $-\frac{1}{2}=m_1\neq m_2=2$. The slopes are unequal so the system has one solution. You found the solution in question 4.

13. The slope for one of the equations is 2. The slope for the other is 1. Since the slopes are unequal, the system has one solution. See question 5 for the solution to the system.

14. $x+2y=-10$ (1)
$x-4y=32$ (2)

From (1): $y=-\frac{1}{2}x-5$

From (2): $y=\frac{1}{4}x-8$

The slopes are unequal so the system has one solution. See question 6 for the solution.

15. $4x=10-2y$ (1)
$y=-2x+5$

$14x+7y=35$ (2)
$y=-2x+5$

The slopes and the y-intercepts are equal.
Therefore, the system has an infinite number of solutions.

16. $y=5x+4$ (1)
$y=0x+5$ (2)

The slopes are unequal so the system has one solution.

17. The slopes are equal and the y-intercepts are unequal. Therefore, the system has no solution.

18. $x+2y=4$ (1)
$y=-\frac{1}{2}x+4$ (2)

From (1): $y=-\frac{1}{2}x+2$

The slopes are equal and the y-intercepts are unequal. It follows that the system has no solutions.

19. $x = 7$ (1)

$2x = 11$ (2)

From (2), $x = \frac{11}{2} = 5\frac{1}{2}$.

Neither equation can be put in the $y = mx + b$ form and the equations have different x-intercepts. Both of these lines are vertical (same slope). Since they have different x-intercepts, there are no solutions.

20. $x + 5y = 15$ (1)

$y = \frac{-1}{5}x + 3$

$x = 10$ (2)

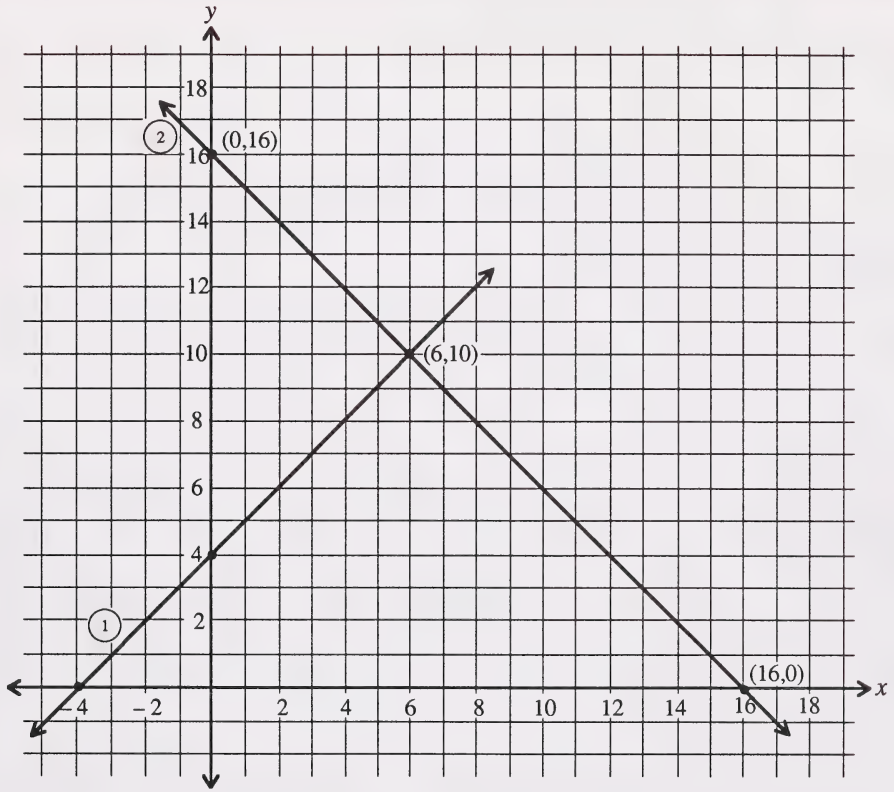
Equation (1) has a slope of $-\frac{1}{5}$ and Equation (2) is vertical.
Since they have different slopes, there is one solution.

21. Let the width be x metres and the length be y metres.

Then $y - x = 4$
and $2x + 2y = 32$.

For (1), the, intercepts are (0, 4) and (– 4, 0).

For (2), the intercepts are (0, 16) and (16, 0).



The graphs intersect at (6, 10).

It follows that the solution to the system is $x = 6$ and $y = 10$.

The length is 10 m and the width is 6 m.

$$P = 2l + 2w$$
$$= 2(10\,\text{m}) + 2(6\,\text{m})$$
$$= 20\,\text{m} + 12\,\text{m}$$
$$= 32\,\text{m}$$

This checks.
The difference between the length and width is 4 m as it should be.

22. Let the time be x hours and the distance travelled be y kilometres.
Then for the boy

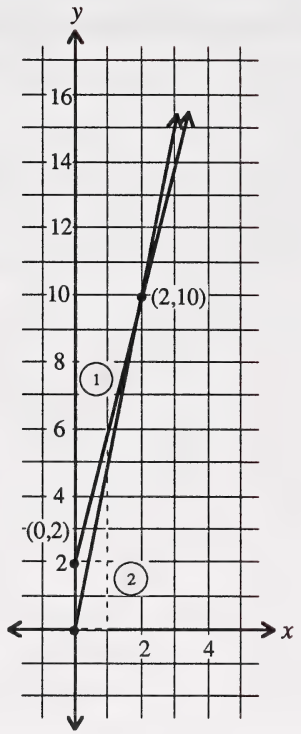
$y = 4x + 2$ (1) and

for the sister

$y = 5x$. (2)

For (1), the slope is 4 and the y-intercept is (0, 2).

For (2), the slope is 5 and the y-intercept is (0, 0).



The graphs intersect at (2, 10).

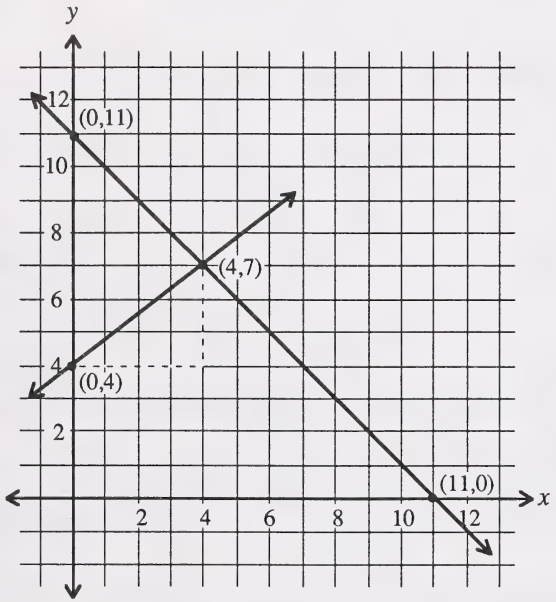
The solution of the system of equations is $x = 2$ and $y = 10$.

It will take the sister 2 h to catch up. Then both will be 10 km down the road.

23. Let the numbers be x and y.
Then $4y-3x=16$ (1)
and $y+x=11$. (2)

Placed in the slope-intercept form, (1) becomes $y=\frac{3}{4}x+4$.
The slope is $\frac{3}{4}$ and the y-intercept is (0, 4).

The intercepts of (2) are (0, 11) and (11, 0).



The graphs cross at (4, 7).

The solution of the system is $x = 4$ and $y = 7$.

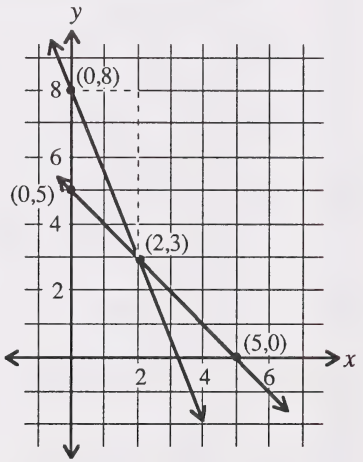
The sum of the numbers is 11. Also 4 times 7 minus 3 times 4 is 16.
This shows that the numbers satisfy the conditions of the problem.

24. Let x be the number of five-dollar bills and
let y be the number of two-dollar bills.

Then the value of the five-dollar bills is $5x$ dollars, the value of the two-dollar bills is $2y$ dollars, and the total number of bills is $x + y$.

Consequently,
$5x+2y=16$ (1)
and $x+y=5$ (2)

In the slope-intercept form, (1) becomes $y=-\frac{5}{2}x+8$.
The slope is $-\frac{5}{2}$ and the y-intercept is (0, 8).
For (2), the intercepts are (0, 5) and (5, 0).



The graphs intersect at (2, 3).

The solution of the system is $x = 2$ and $y = 3$.

There were 2 five-dollar bills and 3 two-dollar bills for a total of five bills.

The value of these 5 bills is 2($5) + 3($2) = $10 + $6 = $16. This satisfies the conditions of the problem.

Activity 2

Solve systems of equations algebraically.

1. $x = -y$ (1)
   $y = x + 3$ (2)

   (1) rearranged: $y = -x$ (3)

   (3) + (2): $y = x + 3$ (2)
   $2y = 3$

   $y = \frac{3}{2}$

   From (1): $x = -y$

   $x = -\left(\frac{3}{2}\right)$

   $x = -\frac{3}{2}$

The solution of the system is $x = -\frac{3}{2}$ and $y = \frac{3}{2}$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$.

Check: $x = -\frac{3}{2}$, $y = \frac{3}{2}$

LS	RS
x	$-y$
$\left(-\frac{3}{2}\right)$	$-\left(\frac{3}{2}\right)$
$-\frac{3}{2}$	$-\frac{3}{2}$

LS = RS

LS	RS
y	$x+3$
$\left(\frac{3}{2}\right)$	$\left(-\frac{3}{2}\right)+3$
$\frac{3}{2}$	$-\frac{3}{2}+\frac{6}{2}$
	$\frac{3}{2}$

LS = RS

2. $2y - 3x = 6$ (1)
   $3x - y = 12$ (2)

   (1) rearranged: $-3x + 2y = 6$ (3)

   (3) + (2): $3x - y = 12$ (2)
   $y = 18$

   From (1): $3x - y = 12$

   $3x - (18) = 12$

   $3x = 30$

   $x = 10$

The solution of the system is $x = 10$ and $y = 18$.

Check: $x = 10$, $y = 18$

LS	RS
$2y - 3x$	6
$2(18) - 3(10)$	
$36 - 30$	
6	

LS = RS

LS	RS
$3x - y$	12
$3(10) - (18)$	
$30 - 18$	
12	

LS = RS

3. $5x+2y=18$ (1)
$4x+4y=24$ (2)

$2 \times$ (1): $10x+4y=36$ (3)
$4x+4y=24$ (2)
(3) − (2): $6x=12$
$x=2$

From (1): $5x+2y=18$
$5(2)+2y=18$
$2y=18-10$
$2y=8$
$y=4$

The solution of the system is (2, 4), that is, $x=2$ and $y=4$.

Check: $x=2$, $y=4$

LS	RS
$5x+2$	18
$5(2)+2(4)$	
$10+8$	
18	

LS = RS

LS	RS
$4x+4y$	24
$4(2)+4(4)$	
$8+16$	
24	

LS = RS

4. $3x+7y=46$ (1)
$5x-9y=-6$ (2)

$5 \times$ (1): $15x+35y=230$ (3)
$3 \times$ (2): $15x-27y=-18$ (4)
$62y=248$
$y=4$

From (1): $3x+7y=46$
$3x+7(4)=46$
$3x+28=46$
$3x=18$
$x=6$

The solution of the system is $x=6$ and $y=4$ or (6, 4).

Check: $x=6$, $y=4$

LS	RS
$3x+7y$	46
$3(6)+7(4)$	
$18+28$	
46	

LS = RS

LS	RS
$5(6)-9(4)$	-6
$30-36$	
-6	

LS = RS

5. $16x+9y=15$ (1)
$6y-4x=-1$ (2)

$16x+9y=15$ (1)

(2) rearranged: $-4x+6y=-1$ (3)

$16x+9y=15$ (1)

$4 \times$ (3): $-16x+24y=-4$ (4)

(1) + (4): $33y=11$

$y=\frac{1}{3}$

From (1): $16x+9y=15$

$16x+9\left(\frac{1}{3}\right)=15$

$16x+3=15$

$16x=15-3$

$16x=12$

$x=\frac{12}{16}$

$x=\frac{3}{4}$

The solution of the system is $x=\frac{3}{4}$ and $y=\frac{1}{3}$.

Check: $x=\frac{3}{4}$, $y=\frac{1}{3}$

LS	RS
$16x+9y$	15
$16\left(\frac{3}{4}\right)+9\left(\frac{1}{3}\right)$	
$12+3$	
15	

LS = RS

LS	RS
$6y-4x$	-1
$6\left(\frac{1}{3}\right)-4\left(\frac{3}{4}\right)$	
$2-3$	
-1	

LS = RS

6. $40x+31y=692$ (1)
$60x-17y=276$ (2)

The L.C.M. of 40 and 60 is 120.

$40=10\times2\times2$

$60=10\times2\times3$

L.C.M. $=10\times2\times2\times3$

$=120$

The constant used to multiply (1) is $\frac{120}{40}=3$.

The constant used to multiply (2) is $\frac{120}{60}=2$.

$3 \times$ (1): $120x+93y=2076$ (3)

$2 \times$ (2): $120x-34y=552$ (4)

(3) − (4): $0+127y=1524$

$y=\frac{1524}{127}$

$y=12$

From (1): $40x+31y=692$

$40x+31(12)=692$

$40x+372=692$

$40x=692-372$

$40x=320$

$x=\frac{320}{40}$

$x=8$

The solution of the system is (8, 12).

Check: $x = 8, y = 12$

LS	RS
$40x + 31y$	692
$40(8) + 31(12)$	
$320 + 372$	
692	

LS = RS

LS	RS
$60x - 17y$	276
$60(8) - 17(12)$	
$480 - 204$	
276	

LS = RS

Check: $x = 6, y = 4$

LS	RS
$32 - 2x$	$5y$
$32 - 2(6)$	$5(4)$
$32 - 12$	20
20	

LS = RS

LS	RS
$3y - 5x$	-18
$3(4) - 5(6)$	
$12 - 30$	
-18	

LS = RS

7. $32 - 2x = 5y$ (1)
$3y - 5x = -18$ (2)

Rearranging (1): $2x + 5y = 32$ (3)
Rearranging (2): $-5x + 3y = -18$ (4)

$5 \times$ (3): $10x + 25y = 160$ (5)
$2 \times$ (4): $-10x + 6y = -36$ (6)
(5) + (6): $31y = 124$
$y = 4$

From (2): $3y - 5x = -18$
$3(4) - 5x = -18$
$12 - 5x = -18$
$-5x = -30$
$x = 6$

The solution of the system is (6, 4).

8. $10a - 3b = 4$ (1)
$21b - 12a = 1$ (2)

Rearranging (2): $10a - 3b = 4$ (1)
$-12a + 21b = 1$ (4)

$7 \times$ (1): $70a - 21b = 28$ (3)
$-12a + 21b = 1$ (4)
(3) + (2): $58a = 29$
$a = \frac{29}{58}$
$a = \frac{1}{2}$

From (2): $21b - 12a = 1$
$21b - 12\left(\frac{1}{2}\right) = 1$
$21b - 6 = 1$
$21b = 7$
$b = \frac{1}{3}$

The solution of the system is $a = \frac{1}{2}$ and $b = \frac{1}{3}$.

Check: $a=\frac{1}{2},\ b=\frac{1}{3}$

LS	RS
$10a-3b$	4
$10\left(\frac{1}{2}\right)-3\left(\frac{1}{3}\right)$	
$5-1$	
4	

LS = RS

LS	RS
$21b-12a$	1
$21\left(\frac{1}{3}\right)-12\left(\frac{1}{2}\right)$	
$7-6$	
1	

LS = RS

9. $13x+36y=-259$ (1)
$17x-24y=301$ (2)

The variable y is chosen for elimination. Its coefficients are 36 and – 24.

The L.C.M. of – 24 and 36 is 72.

$36=\boxed{3}\times\boxed{3}\times\boxed{2}\times\boxed{2}$
$24=\quad\boxed{3}\times\boxed{2}\times\boxed{2}\times\boxed{2}$
$\text{L.C.M.}=\boxed{3}\times\boxed{3}\times\boxed{2}\times\boxed{2}\times\boxed{2}$
$=72$

The constant used to multiply (1) is $\frac{72}{36}=2$.

The constant used to multiply (2) is $\frac{72}{24}=3$.

$2\times(1)$: $26x+72y=-518$ (3)
$3\times(2)$: $51x-72y=903$ (4)
(3) + (4): $77x=385$
$x=5$

From (1): $13x+36y=-259$
$13(5)+36y=-259$
$65+36y=-259$
$36y=-324$
$y=-9$

The solution of the system is $x=5$ and $y=-9$.

Check: $x=5,\ y=-9$

LS	RS
$13x+36y$	– 259
$13(5)+36(-9)$	
$65+(-324)$	
– 259	

LS = RS

LS	RS
$17x-24y$	301
$17(5)-24(-9)$	
$85+216$	
301	

LS = RS

10. $5m-3n=5$ (1)
$2m+9n=2$ (2)

$3\times(1)$: $15m-9n=15$ (3)
$2m+9n=2$ (2)
(3) + (2): $17m=17$
$m=1$

From (1): $5m-3n=5$
$5(1)-3n=5$
$5-3n=5$
$-3n=0$
$n=0$

The solution of the system is $m=1$ and $n=0$.

Check: $m = 1, n = 0$

LS	RS
$5m - 3n$	5
$5(1) - 3(0)$	
$5 - 0$	
5	

LS = RS

LS	RS
$2m + 9n$	2
$2(1) + 9(0)$	
$2 + 0$	
2	

LS = RS

11. $2y = x - 5$ (1)
$2x = y + 3$ (2)

(1) rearranged: $-x + 2y = -5$ (3)
(2) rearranged: $2x - y = 3$ (4)

$2 \times$ (3): $-2x + 4y = -10$ (5)
$2x - y = 3$ (4)

(5) + (4): $3y = -7$

$y = -\frac{7}{3}$

From (2): $2x = y + 3$

$2x = \left(-\frac{7}{3}\right) + 3$

$2x = -\frac{7}{3} + \frac{9}{3}$

$2x = \frac{2}{3}$

$x = \frac{1}{3}$

The solution of the system is $x = \frac{1}{3}$ and $y = -\frac{7}{3}$.

Check: $x = \frac{1}{3}$, $y = -\frac{7}{3}$

LS	RS
$2y$	$x - 5$
$2\left(-\frac{7}{3}\right)$	$\frac{1}{3} - 5$
$-\frac{14}{3}$	$\frac{1}{3} - \frac{15}{3}$
	$-\frac{14}{3}$

LS = RS

LS	RS
$2x$	$y + 3$
$2\left(\frac{1}{3}\right)$	$\left(-\frac{7}{3}\right) + 3$
$\frac{2}{3}$	$-\frac{7}{3} + \frac{9}{3}$
	$\frac{2}{3}$

LS = RS

12. $h = 15 - 2t$ (1)
$h = t$ (2)

Substitute $h = t$ into equation (1).

$t = 15 - 2t$

$3t = 15$

$t = 5$

The number is 5.

Check: $h = 5, t = 5$

LS	RS
h	$15 - 2t$
5	$15 - 2(5)$
	$15 - 10$
	5

LS = RS

LS	RS
h	t
5	5

LS = RS

13. $4x-16y=3$ (1)
$x-2y=0$ (2)

From (2): $x=2y$

Substituting for x in (1) and solving for y:

$$\begin{aligned} 4(2y)-16y&=3\\ 8y-16y&=3\\ -8y&=3\\ y&=-\frac{3}{8}\end{aligned}$$

From (1): $x-2\left(-\frac{3}{8}\right)=0$

$$\begin{aligned} x&=2\left(-\frac{3}{8}\right)\\ x&=-\frac{3}{4}\end{aligned}$$

The solution of the system is $x=-\frac{3}{4}$ and $y=-\frac{3}{8}$.

Check: $x=-\frac{3}{4},\ y=-\frac{3}{8}$

LS	RS
$4x-16y$	3
$4\left(-\frac{3}{4}\right)-16\left(-\frac{3}{8}\right)$	
$-3+6$	
3	

LS = RS

LS	RS
$x-2y$	0
$\left(-\frac{3}{4}\right)-2\left(-\frac{3}{8}\right)$	
$-\frac{3}{4}+\frac{3}{4}$	
0	

LS = RS

14. $5m+2n=44$ (1)
$3m+n=24$ (2)

From (2): $n=24-3m$

Substituting for n in (1) and solving for m:

$$\begin{aligned} 5m+2(24-3m)&=44\\ 5m+48-6m&=44\\ 48-m&=44\\ -m&=-4\\ m&=4\end{aligned}$$

From (2) and substituting the value for m:

$$\begin{aligned} 3(4)+n&=24\\ 12+n&=24\\ n&=12\end{aligned}$$

The solution of the system is $m=4$ and $n=12$.

Check: $m=4,\ n=12$

LS	RS
$5m+2n$	44
$5(4)+2(12)$	
$20+24$	
44	

LS = RS

LS	RS
$3m+n$	24
$3(4)+12$	
$12+12$	
24	

LS = RS

15. $20p+6q=3$ (1)

$2p=-q$ (2)

From (2): $q=-2p$

Substituting for q in (1) and solving for p:

$$20p+6(-2p)=3$$
$$20p-12p=3$$
$$8p=3$$
$$p=\frac{3}{8}$$

Substituting the value for p in (2):

$$2\left(\frac{3}{8}\right)=-q$$
$$\frac{3}{4}=-q$$
$$q=-\frac{3}{4}$$

The solution of the system is $p=\frac{3}{8}$ and $q=-\frac{3}{4}$.

Check: $p=\frac{3}{8}$, $q=-\frac{3}{4}$

LS	RS
$20p+6q$	3
$20\left(\frac{3}{8}\right)+6\left(-\frac{3}{4}\right)$	
$\frac{60}{8}-\frac{36}{8}$	
$\frac{24}{8}$	
3	

LS = RS

LS	RS
$2p$	$-q$
$2\left(\frac{3}{8}\right)$	$-\left(-\frac{3}{4}\right)$
$\frac{3}{4}$	$\frac{3}{4}$

LS = RS

16. $10x+2y=-1$ (1)

$2x=1-y$ (2)

From (2): $y=1-2x$

Substitute for y in (1) and solve for x.

$$10x+2(1-2x)=-1$$
$$10x+2-4x=-1$$
$$6x+2=-1$$
$$6x=-3$$
$$x=-\frac{1}{2}$$

Substitute the value for x in (2) and solve for y.

$$2\left(-\frac{1}{2}\right)=1-y$$
$$-1=1-y$$
$$-2=-y$$
$$y=2$$

The solution of the system is $x=-\frac{1}{2}$ and $y=2$.

Check: $x=-\frac{1}{2}$, $y=2$

LS	RS
$10x+2$	-1
$10\left(-\frac{1}{2}\right)+2(2)$	
$-5+4$	
-1	

LS = RS

LS	RS
$2x$	$1-y$
$2\left(-\frac{1}{2}\right)$	$1-2$
-1	-1

LS = RS

17. $x+y=10$ (1)
$x-y=10$ (2)

From (1): $y=10-x$

Substituting for y in (2) and solving for x:

$$x-(10-x)=10$$
$$2x-10=10$$
$$2x=20$$
$$x=10$$

From (1): $(10)+y=10$
$$y=0$$

The solution of the system is (10, 0) or $x = 10$ and $y = 0$.

Check: $x = 10, y = 0$

LS	RS
$x+y$	10
$10+(0)$	
10	

LS = RS

LS	RS
$x-y$	10
$10-(0)$	
10	

LS = RS

18. $x+\frac{4}{5}y=2$ (1)
$5x+4y=10$ (2)

From (1): $x=-\frac{4}{5}y+2$

Substitute the equivalent expression for x in (2) and solve for x.

$$5\left(-\frac{4}{5}y+2\right)+4y=10$$
$$(-4y+10)+4y=10$$
$$10=10$$

The last equation is true and has no variable. It follows that the system has an infinite number of solutions.

Some sample solutions are (– 2, 5), (2, 0), (6, – 5), (10, – 10), and (14, – 15).

19. $3y-2x=5$ (1)
$3y-2x=1$ (2)
(1) – (2): $0=4$, which is a false statement with no variables.

The system has no solution.

20. Let the numbers be x and y.

Then $x+y=100$ (1)

and $x-y=124.$ (2)

(1) + (2): $2x=224$

$x=112$

From (1): $(112)+y=100$

$y=-12$

The solution of the system is $x = 112$ and $y = -12$.

The numbers are 112 and -12.

Check:

The sum of these numbers is 100 and their difference, $112-(-12)$, is 124. These numbers satisfy the problem conditions.

21. Let the numbers be n and m.

Then $n+m=165$ (1)

and $3n+5m=595.$ (2)

$3\times$ (1): $3n+3m=495$ (3)

$3n+5m=595$ (2)

$-2m=-100$

$m=50$

From (1): $n+50=165$

$n=115$

The solution of the system is $n = 115$ and $m = 50$.

The numbers are 115 and 50.

Check:

The sum of these numbers is 165. Also three times 115 plus five times 50 is 345 + 250 = 595. The numbers satisfy the conditions in the problem.

22. Let the number of dimes be x and the number of quarters be y.

Then the value of the dimes is $10x$ cents, the value of the quarters is $25y$ cents, and the total number of coins is $x + y$.

$10x+25y=625$ (1)

$x+y=46$ (2)

$10x+25y=625$ (1)

$10\times$ (2): $10x+10y=460$ (3)

(1) − (3): $15y=165$

$y=11$

From (2): $x+(11)=46$

$x=46-11$

$x=35$

The solution of the system is $x = 35$ and $y = 11$.

Bill Hroshee has 35 dimes and 11 quarters in his pocket.

Check:

Bill has 46 coins in his pocket when there are 35 dimes and 11 quarters. The total value of these coins is 35($0.10) + 11($0.25) = $3.50 + $2.75 = $6.25. This verifes the solution to the problem.

23. Let the numbers be x and y.

Then $x+y=601.$ (1)

$x-y=89$ (2)

(1) + (2): $2x=690$

$x=345$

From (1): $(345)+y=601$

$y=256$

The solution of the system is $x = 345$ and $y = 256$.

Check:

The numbers are 345 and 256. Their sum is 345 + 256 = 601 and their difference is 345 – 256 = 89 as required.

24. Let the number of bull's-eyes be x and the number of outer zones be y.

Then $10x+6y=62.$ (1)

$x+y=9$ (2)

$10\times$(2): $10x+10y=90$ (3)

$10x+6y=62$ (1)

(3) – (1): $4y=28$

$y=7$

From (1): $x+(7)=9$

$x=2$

The solution of the system is $x = 2$ and $y = 7$.

The contestant makes 2 bull's-eyes and 7 outer zones.

Check:

The contestant makes a total of 9 shots for a total of 2(10 points) + 7(6 points) = 20 points + 42 points = 62 points.

25. Let the number of ten-dollar bills be x and the number of twenty-dollar bill be y.

Then $x+y=19$ (1)

and $10x+20y=350.$ (2)

$10x+20y=350$ (2)

$10\times$(1): $10x+10y=190$ (3)

$10y=160$

$y=16$

From (1): $x+(16)=19$

$x=3$

The solution of the system is $x = 3$ and $y = 16$.

Frances received 3 ten-dollar bills and 16 twenty-dollar bills.

Check:

Frances received a total of 3 bills + 16 bills = 19 bills. The total value of these bills is 3($10) + 16($20) = $30 + $320 = $350. The solution is verified.

26. Let the numbers be n and m.

Then $n+m=54$ (1)

and $5n-2m=109.$ (2)

$2\times(1)$: $2n+2m=108$ (3)

$5n-2m=109$ (2)

(3) + (2): $7n=217$

$n=31$

From (1): $(31)+m=54$

$m=54-31$

$m=23$

The solution of the system is 23 and 31.

The numbers are 23 and 31.

Check:

The sum of the numbers is 23 + 31 = 54. As well, the difference between five times 31 and two times 23 is 5(31) – 2(23) = 155 – 46 = 109.

Therefore, the solution is correct.

27. Let the cost of a ham sandwich be x dollars and the cost of a cheese sandwich be y dollars.

Then $7x+5y=21.50$ (1)

and $5x+7y=20.50.$ (2)

$5\times(1)$: $35x+25y=107.50$ (3)

$7\times(2)$: $35x+49y=143.50$ (4)

(3) – (4): $-24y=-36$

$y=1.50$

From (1): $7x+5(1.50)=21.50$

$7x+7.50=21.50$

$7x=14.00$

$x=2.00$

The solution of the system is x = 2.00 and y = 1.50.

A ham sandwich costs $2.00 and a cheese sandwich costs $1.50.

Check:

Seven ham sandwiches and five cheese sandwiches cost 7($2.00) + 5($1.50) = $14.00 + $7.50 = $21.50.

Five ham sandwiches and seven cheese sandwiches cost 5($2.00) + 7($1.50) = $10.00 + $10.50 = $20.50.

This verifies the answer to the problem.

Extra Help

1. $x+3y=6$ (1)
   $x+y=4$ (2)

For (1): When $x=0$,

$x+3y=6.$

$(0)+3y=6$

$y=2$

The y-intercept is (0, 2).

When $y=0$,

$x+3y=6.$

$x+3(0)=6$

$x=6$

The x-intercept is (6, 0).

For (2): When $x=0$,

$x+y=4.$

$(0)+y=4$

$y=4$

The y-intercept is (0, 4).

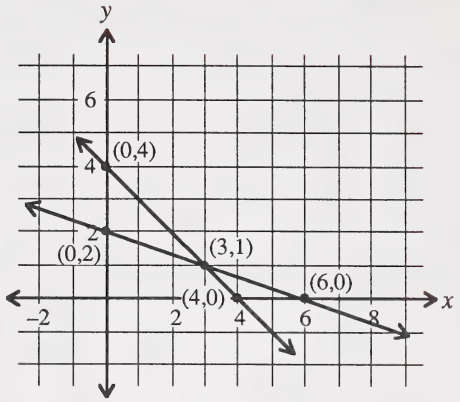
When $y=0$,

$x+y=4.$

$x+(0)=4$

$x=4$

The y-intercept is (4, 0).



The graphs cross at (3, 1).

The solution of the system is $x = 3$ and $y = 1$.

Check: $x = 3, y = 1$

LS	RS
$x + 3y$	6
$3 + 3(1)$	
6	

LS = RS

LS	RS
$x + y$	4
$(3) + (1)$	
4	

LS = RS

2. $21x-3y=1$ (1)
$3y-21x=1$ (2)

$21x-3y=1$ (1)
$-3y=-21x+1$
$y=7x-\frac{1}{3}$

$3y-21x=1$ (2)
$3y=21x+1$
$y=7x+\frac{1}{3}$

The slope for both equations are equal to 7. For (1), the y-intercept is $-\frac{1}{3}$, and for (2), it is $+\frac{1}{3}$.
The equations have equal slopes and y-intercepts that are unequal. Therefore, the system has no solution.

3. $31x-y=-10$ (1)
$17x+y=8$ (2)

$31x-y=-10$ (1)
$-y=-31x-10$
$y=31x+10$

$17x+y=8$ (2)
$y=-17x+8$

For (1), the slope is 31, and for (2), it is – 17. Since the slopes of the equations are unequal, the system has one solution.

4. $2x-3y=0$ (1)
$5x+3y=21$ (2)

(1) + (2): $7x+0=21$
$7x=21$
$x=3$

From (1): $2x-3y=0$
$2(3)-3y=0$
$6-3y=0$
$-3y=-6$
$y=2$

The solution of the system is $x = 3$ and $y = 2$.

Check: $x = 3, y = 2$

LS	RS
$2x-3y$	0
$2(3)-3(2)$	
$6-6$	
0	

LS = RS

LS	RS
$5x+3y$	21
$5(3)+3(2)$	
$15+6$	
21	

LS = RS

5. $3x+4y=5$ (1)
$2x+7y=12$ (2)

$2 \times (1)$: $6x+8y=10$ (3)
$3 \times (2)$: $6x+21y=36$ (4)
(3) − (4): $-13y=-26$
$y=2$

From (1): $3x+4y=5$
$3x+4(2)=5$
$3x+8=5$
$3x=-3$
$x=-1$

The solution of the system is $x=-1$ and $y=2$.

Check: $x=-1, y=2$

LS	RS
$3x+4y$	5
$3(-1)+4(2)$	
$-3+8$	
5	

LS = RS

LS	RS
$2x+7y$	12
$2(-1)+7(2)$	
$-2+14$	
12	

LS = RS

Extensions

1. $a+b+0=2$ (1)
$a+0+c=7$ (2)
$0+b+c=-1$ (3)

$a+b+0=2$ (1)
$a+0+c=7$ (2)
(1) − (2): $0+b-c=-5$ (4)
$0+b+c=-1$ (3)
(4) + (3): $2b+0=-6$
$2b=-6$
$b=-3$

From (1): $a+b+0=2$
$a+(-3)+0=2$
$a=5$

From (3): $0+b+c=-1$
$0+(-3)+c=-1$
$c=2$

The solution to the system is $a=5$, $b=-3$, and $c=2$.

Check: $a=5, b=-3, c=2$

LS	RS
$a+b$	2
(5) + (−3)	
2	

LS = RS

LS	RS
$a+c$	7
(5) + (2)	
7	

LS = RS

LS	RS
$b+c$	−1
(−3) + (2)	
−1	

LS = RS

2. $2x+2y+z=3$ (1)
$x+y+0=1$ (2)
$3x+2y+0=0$ (3)

$2x+2y+z=3$ (1)
$2 \times$ (2): $2x+2y+0=2$ (4)
(1) − (4): $0+0+z=1$
$z=1$

$3 \times$ (2): $3x+3y+0=3$ (5)
$3x+2y+0=0$ (3)
(5) − (3): $0+y+0=3$
$y=3$

From (2): $x+y+0=1$
$x+(3)+0=1$
$x+3=1$
$x=-2$

The solution to the system is $x=-2$, $y=3$, and $z=1$.

Check: $x=-2$, $y=3$, $z=1$

LS	RS
$2x+2y+z$	3
$2(-2)+2(3)+(1)$	
$-4+6+1$	
3	

LS = RS

LS	RS
$x+y$	1
$(-2)+(3)$	
1	

LS = RS

LS	RS
$3x+2y$	0
$3(-2)+2(3)$	
$-6+6$	
0	

LS = RS

3. $x+y+z=-3$ (1)
$2x+3y-z=-13$ (2)
$4x-5y+3z=-5$ (3)

$x+y+z=-3$ (1)
$2x+3y-z=-13$ (2)
(1) + (2): $3x+4y=-16$ (4)

$3 \times$ (2): $6x+9y-3z=-39$ (5)
$4x-5y+3z=-5$ (3)
(5) + (3): $10x+4y=-44$ (6)

$10x+4y=-44$ (6)
$3x+4y=-16$ (4)
(6) − (4): $7x=-28$
$x=-4$

From (4): $3x+4y=-16$
$3(-4)+4y=-16$
$-12+4y=-16$
$4y=-4$
$y=-1$

From (1): $x+y+z=-3$
$(-4)+(-1)+z=-3$
$-5+z=-3$
$z=2$

The solution to the system is $x=-4$, $y=-1$, and $z=2$.

Check: $x = -4, y = -1, z = 2$

LS	RS
$x + y + z$	-3
$(-4) + (-1) + (2)$	
-3	

LS = RS

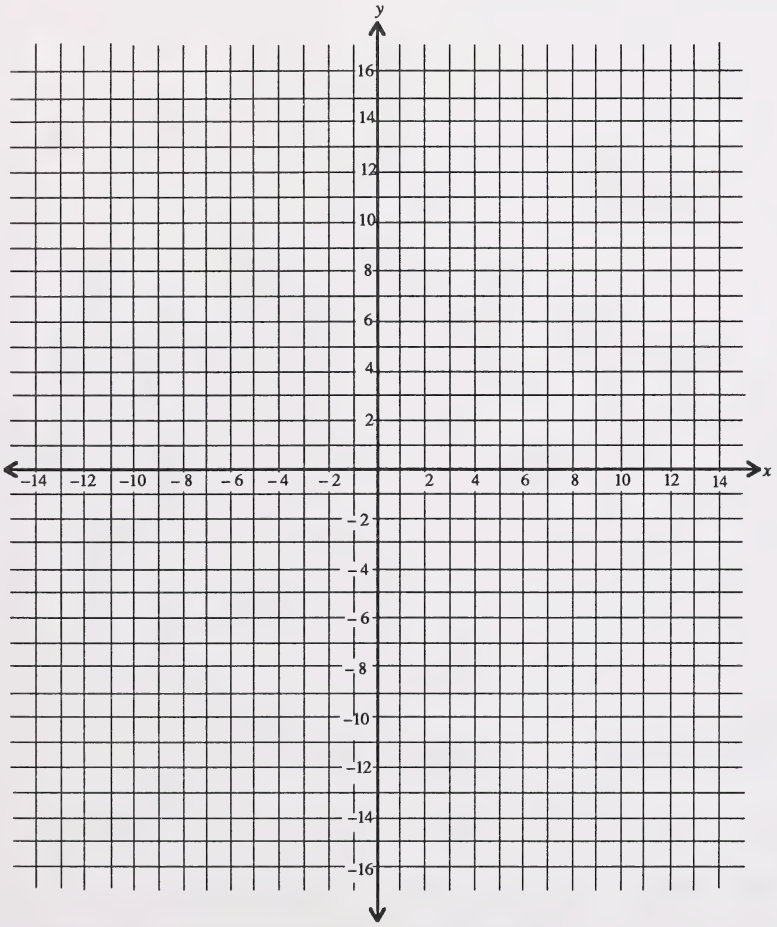
LS	RS
$2x + 3y - z$	-13
$2(-4) - 3(-1) - 2$	
$-8 - 3 - 2$	
-13	

LS = RS

LS	RS
$4x - 5y + 3z$	-5
$4(-4) - 5(-1) + 3(2)$	
$-16 + 5 + 6$	
-5	

LS = RS

Appendix B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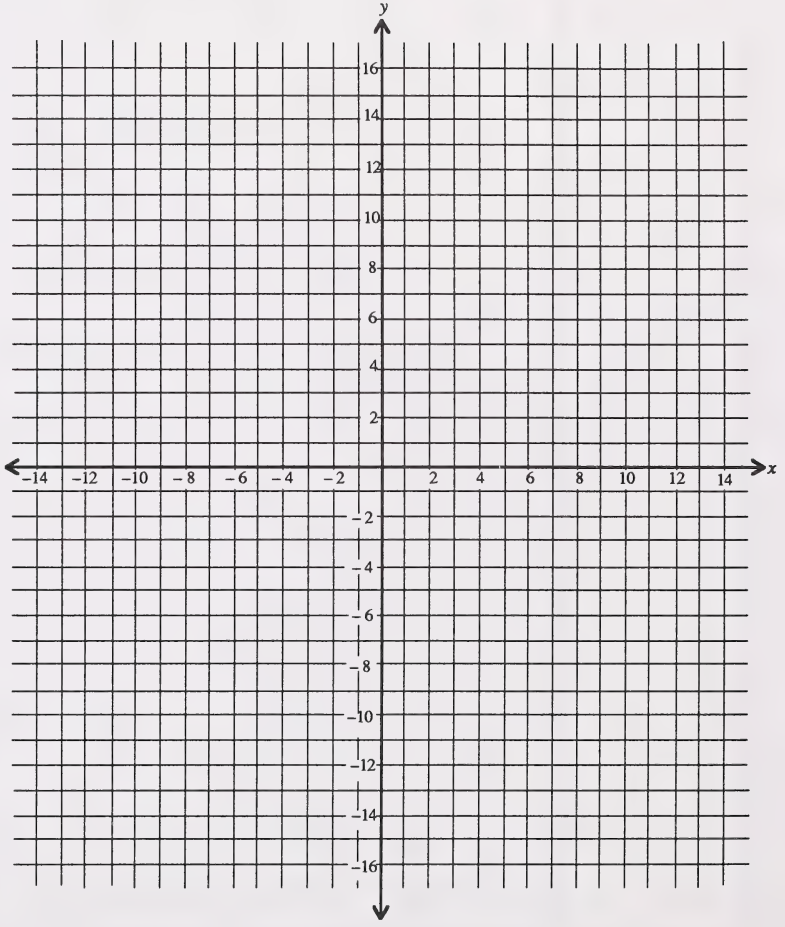
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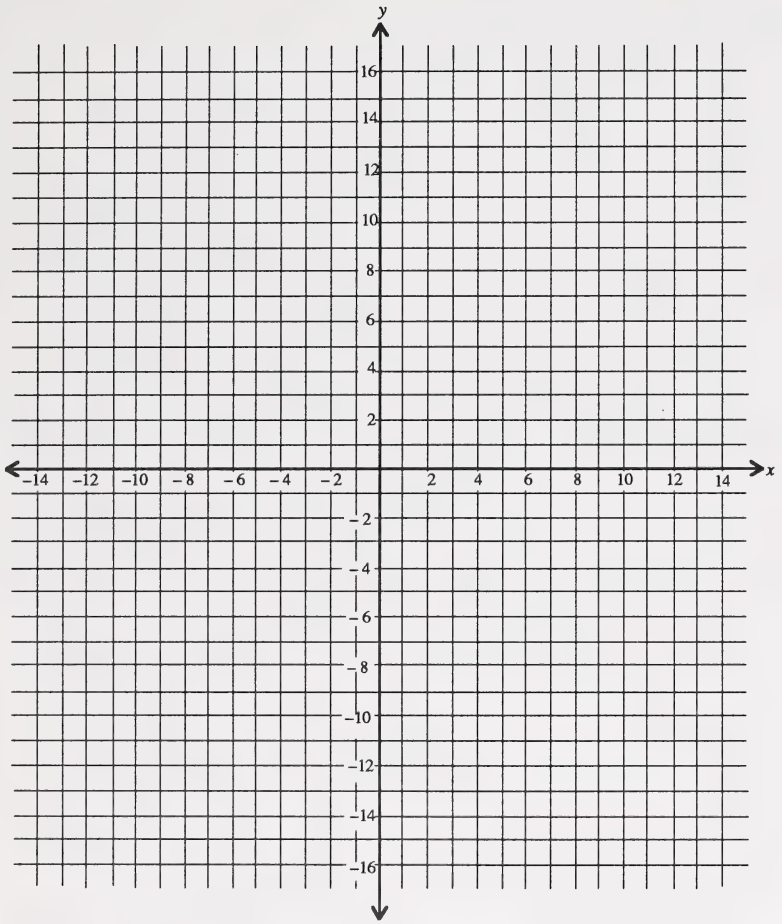
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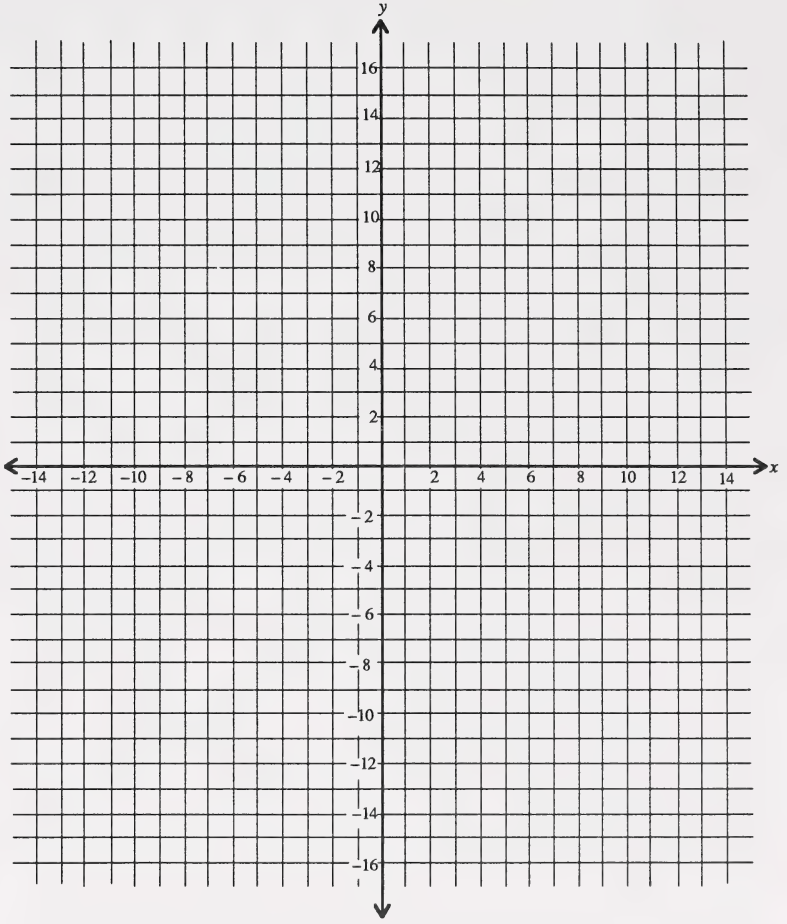
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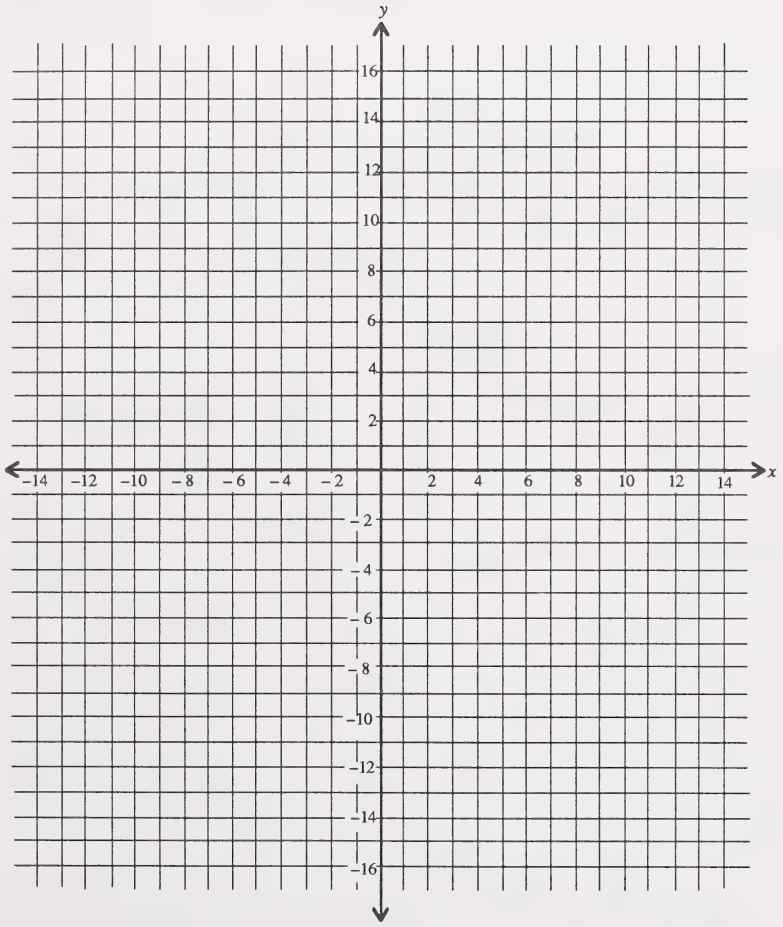
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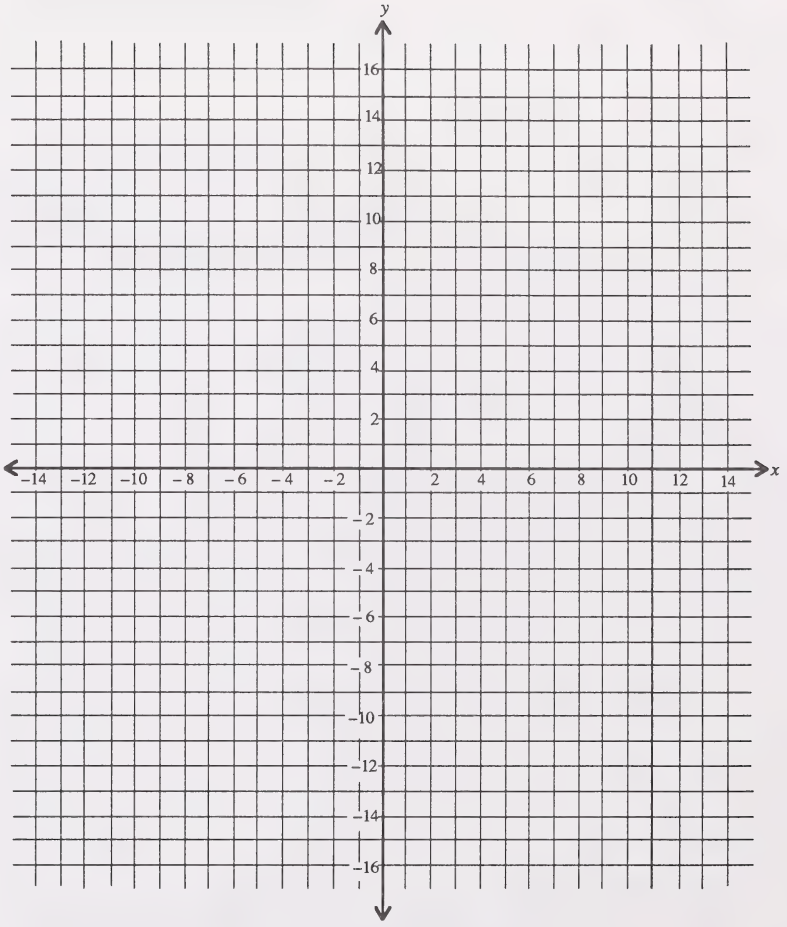
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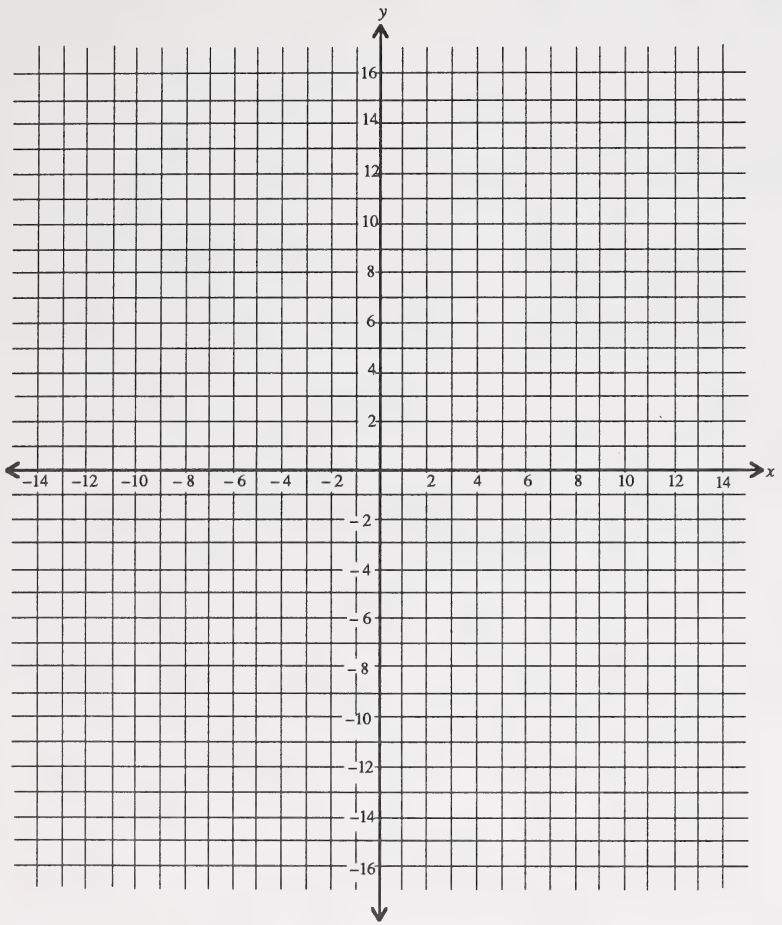
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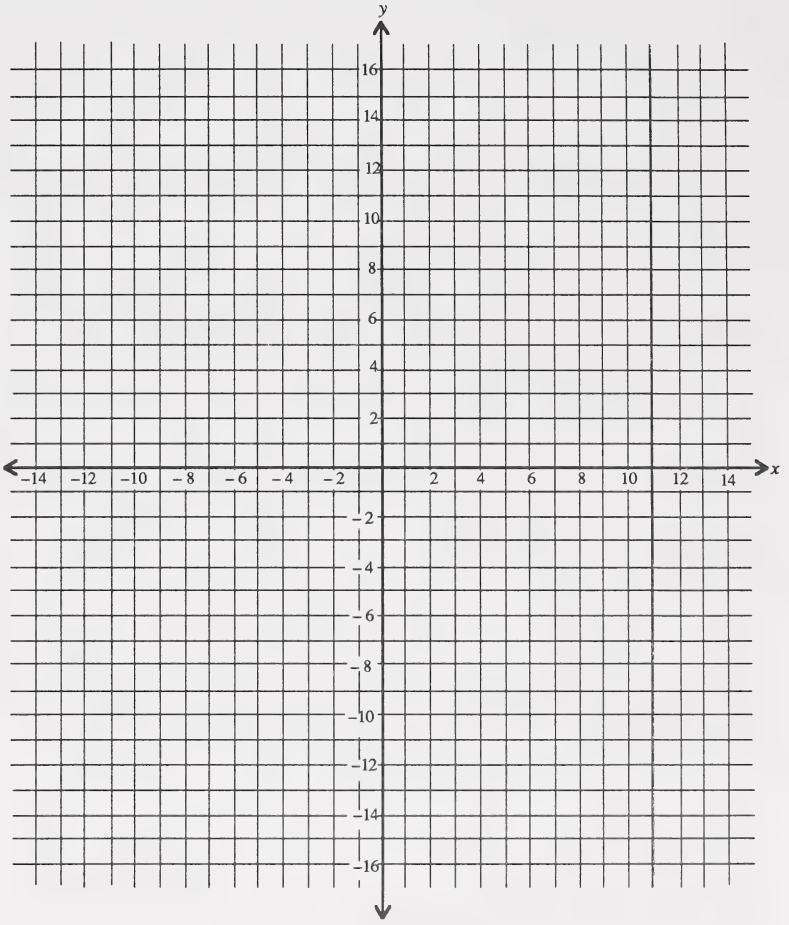
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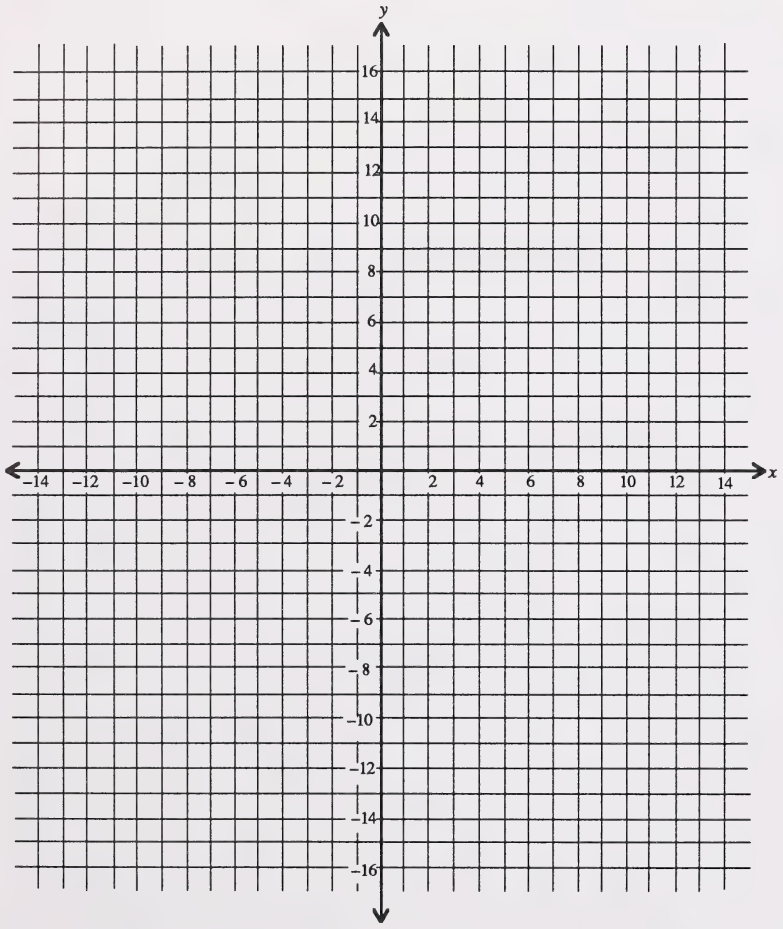
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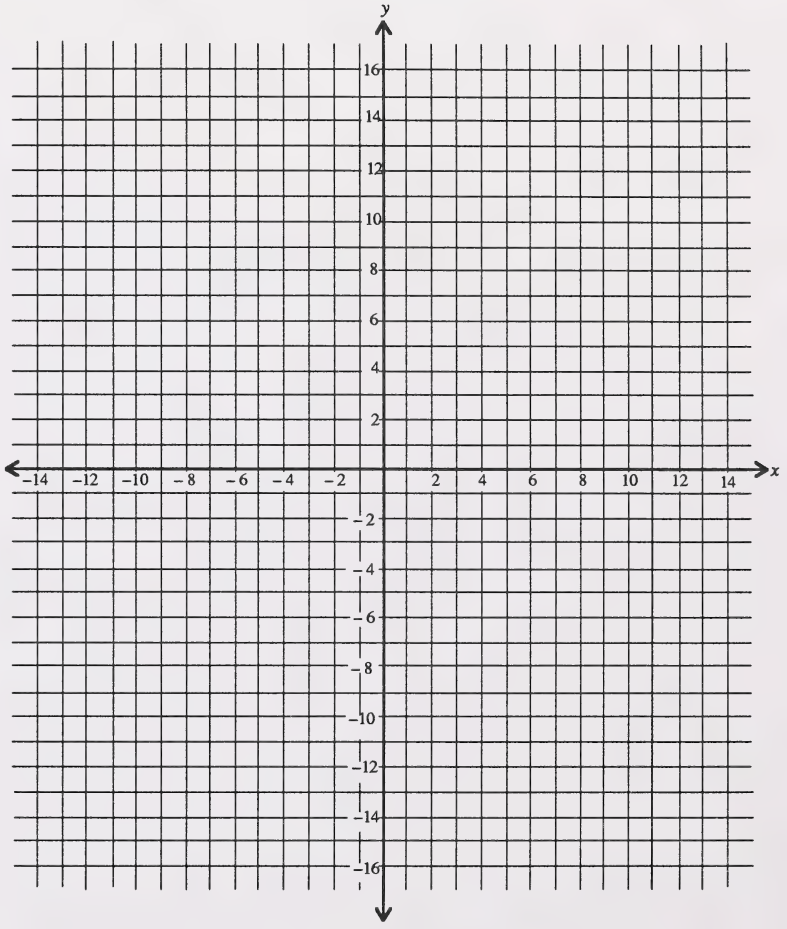
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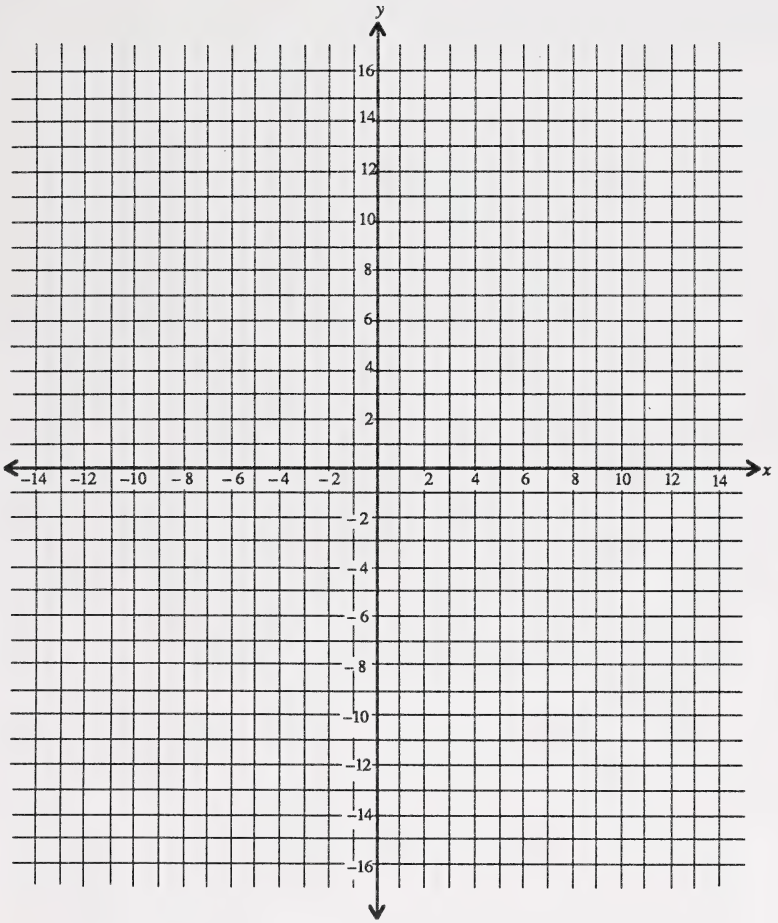
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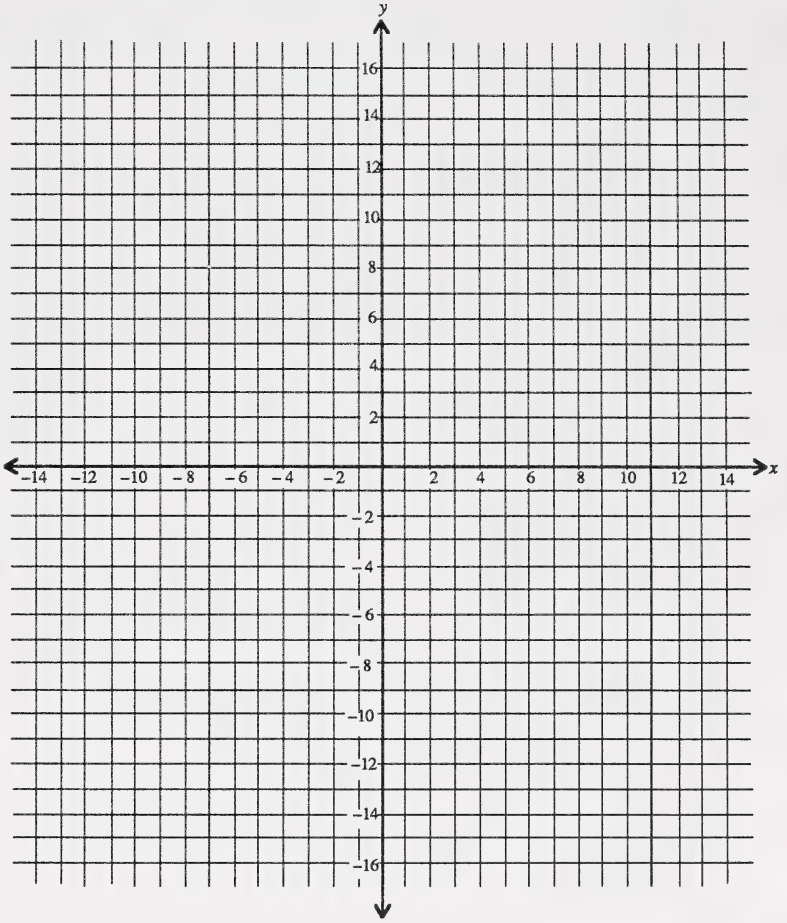
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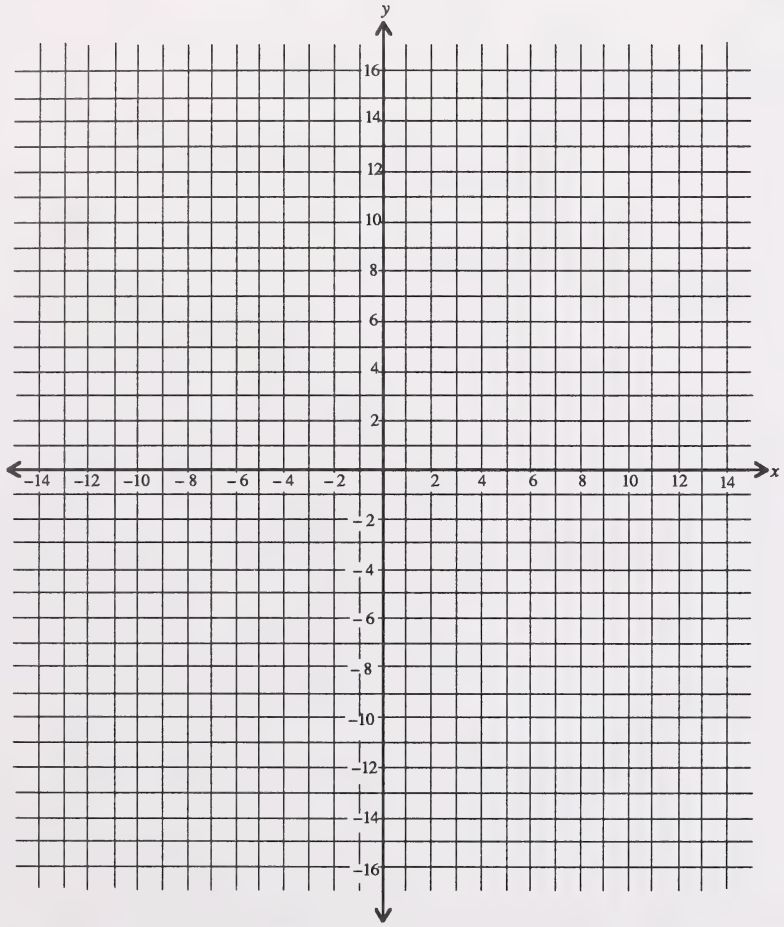
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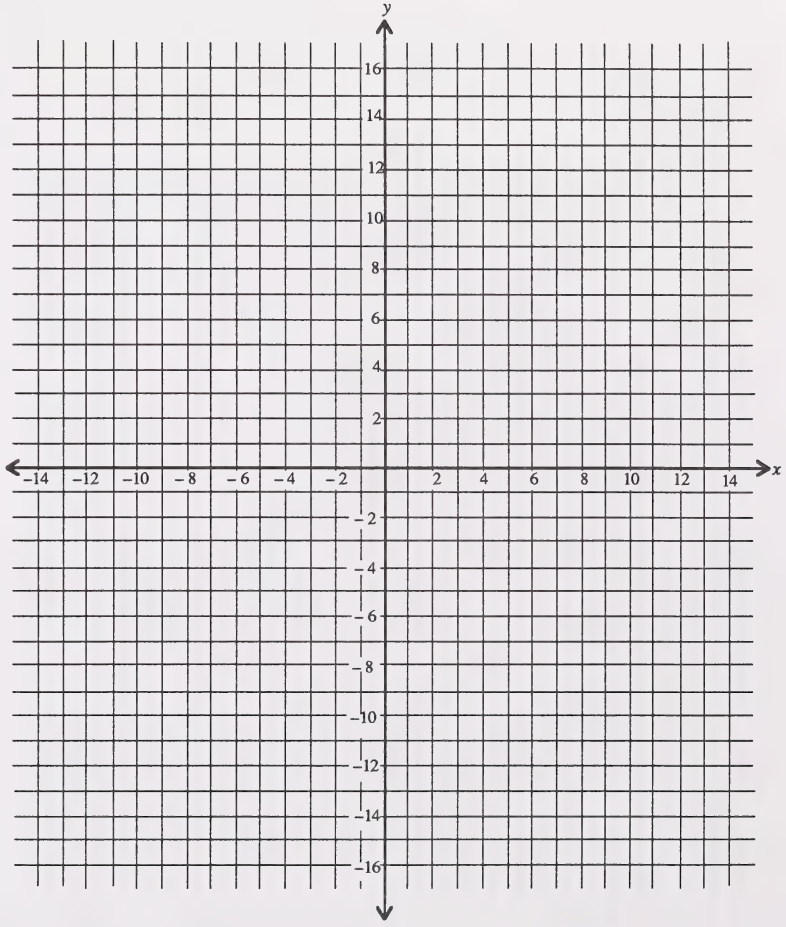
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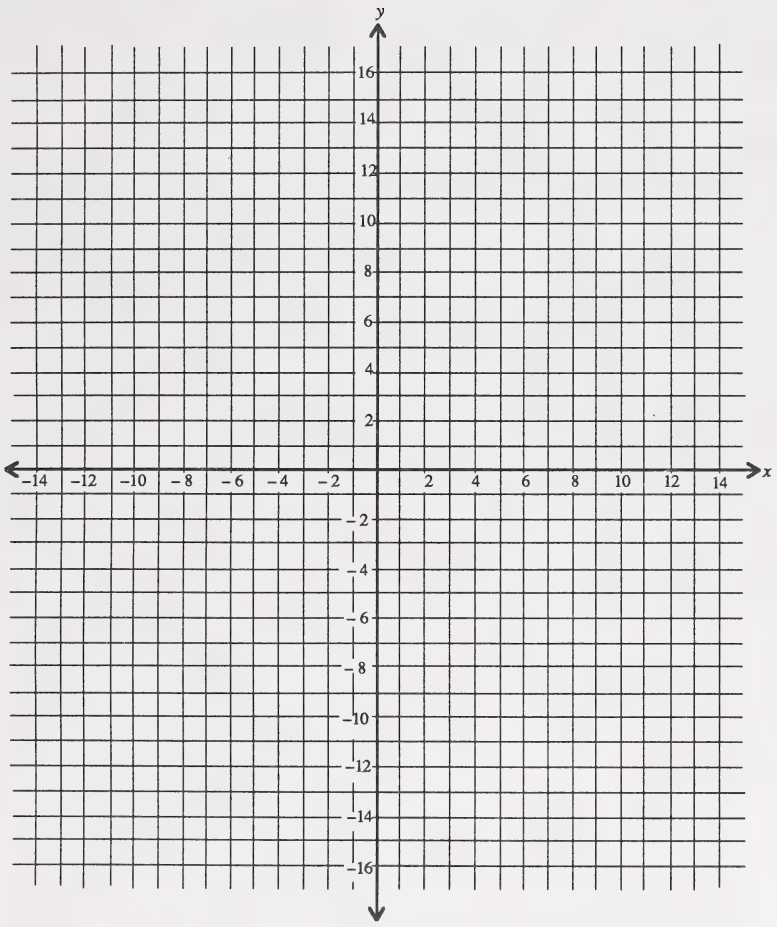
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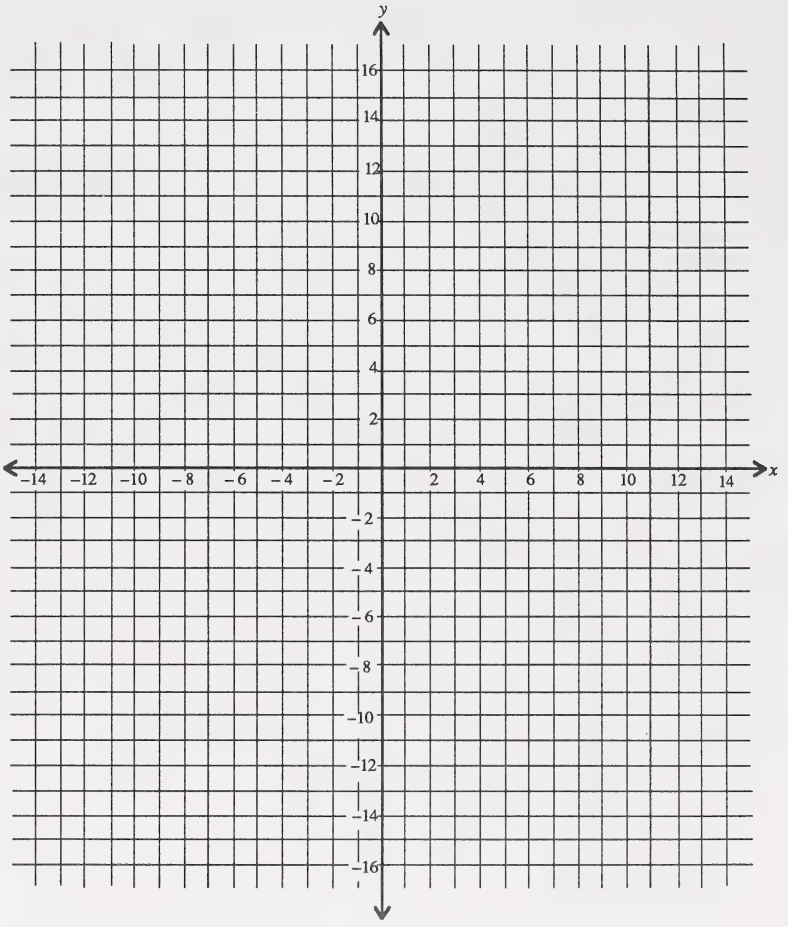
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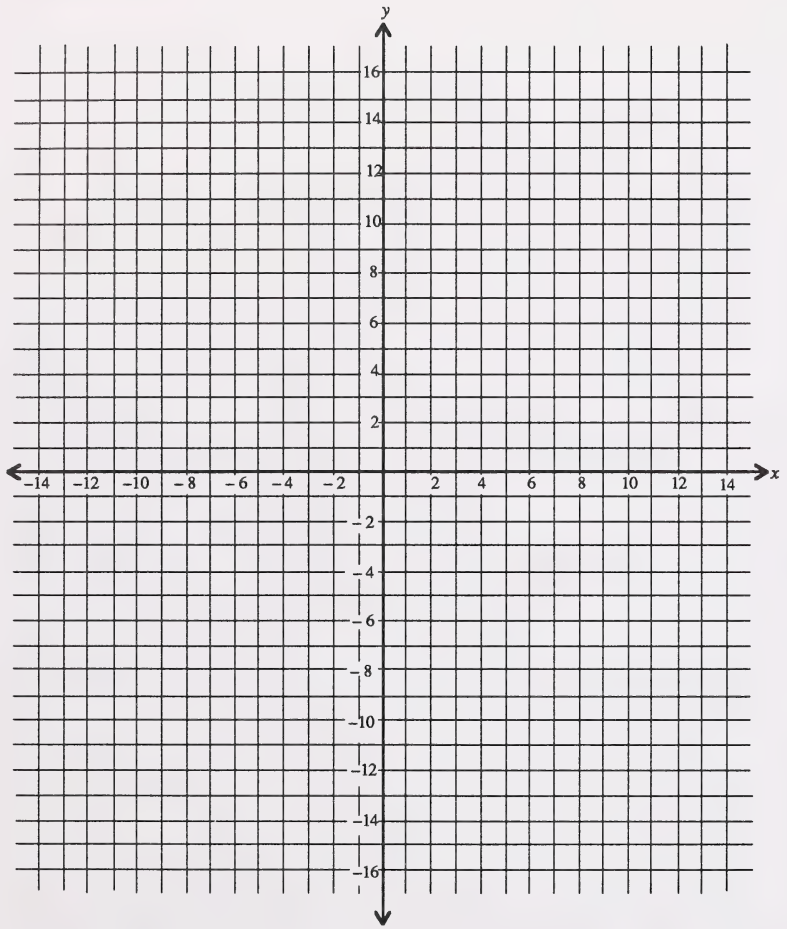
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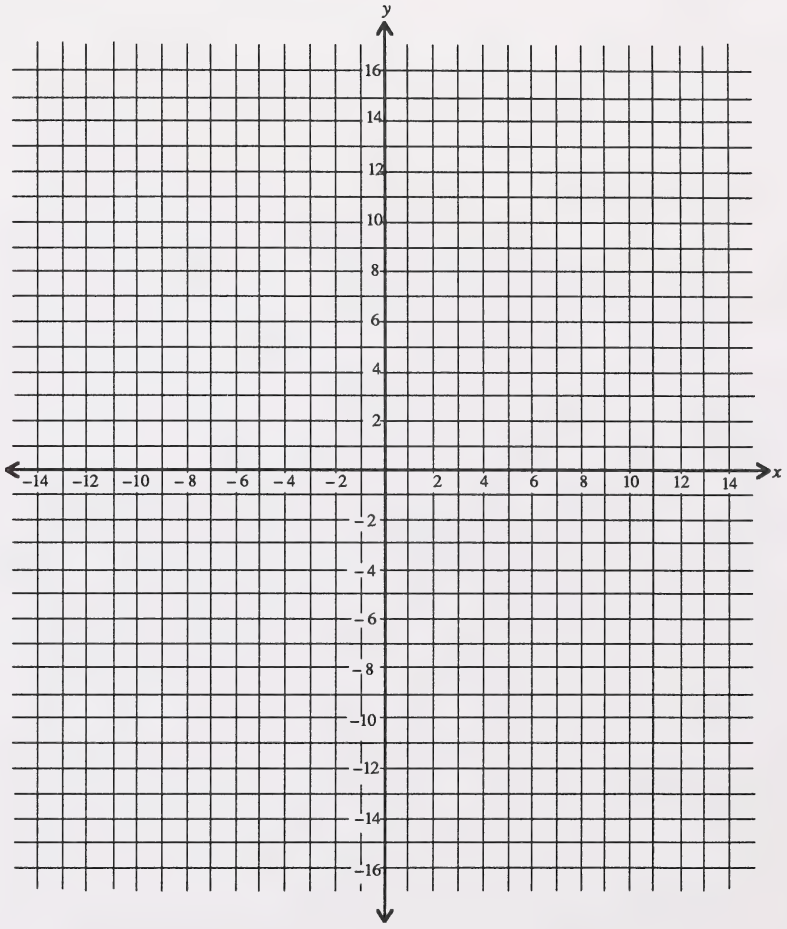
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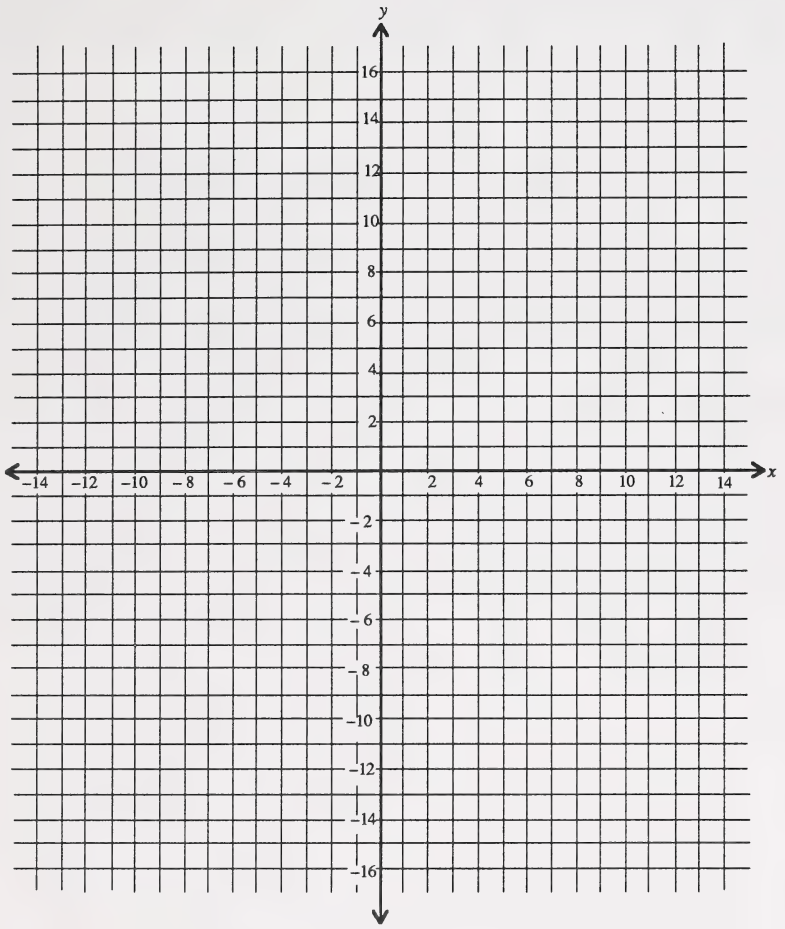
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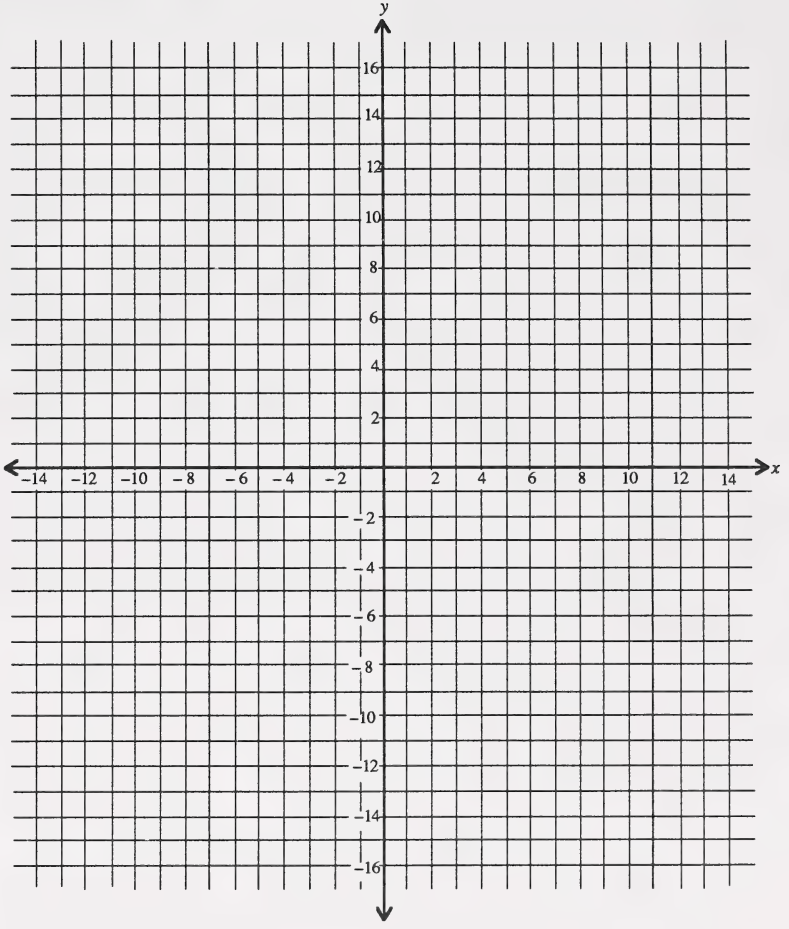
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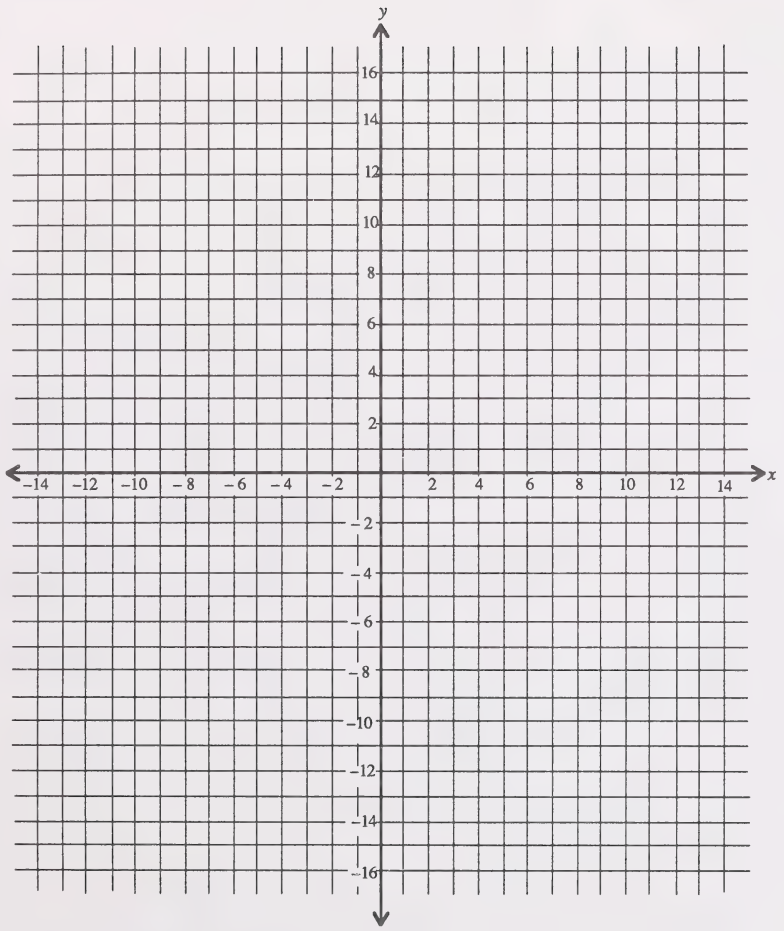
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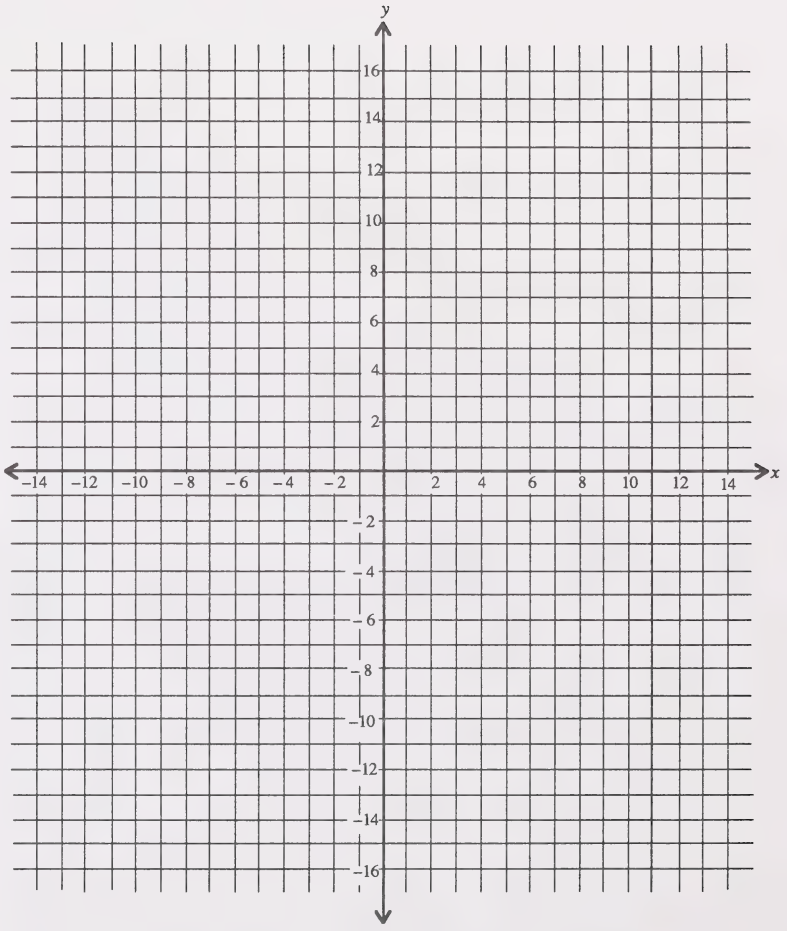
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